MAC 1140 Quadratic Functions Section 3.1 (notes and in-class problems)

Quadratic functions can be written in the form: $f(x)=ax^2+bx+c$, where $a,b,c\in R$, $a\neq 0$.

Vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

They can also be written in the form: $f(x)=a(x-h)^2+k$, where $a,h,k\in R$, $a\neq 0$.

Vertex: (h, k)

In either case, the graph of a Quadratic Function is a PARABOLA.

$a>0 \Rightarrow$ the parabola opens upward

$a<0 \Rightarrow$ the parabola opens downward

There are 3 ways to find the vertex of the parabola. You should be able to do all three.

Example 1: $f(x)=x^2+6x+3$

1) Find the vertex by using $x=\frac{-b}{2a}$: $x=\frac{-b}{2a}=\frac{-6}{2(1)}=-3$ $y=f(-3)=(-3)^2+6(-3)+3=-6$

So, the vertex is $(-3,-6)$.

2) Find the vertex by completing the square:

$f(x)=(x^2+6x)+3$

$f(x)=(x^2+6x+9)+3-9$

$f(x)=(x+3)^2-6$ *Comparing this to the form:* $f(x)=a(x-h)^2+k$ *You can see that the vertex is* $(-3,-6)$.

3) Graph the function with your calculator and find the vertex by using the "min" feature in the "calc" menu. (2nd trace)

Example 2: $f(x)=-2x^2+8x-4$

1) Find the vertex by using $x=\frac{-b}{2a}$: $x=\frac{-b}{2a}=\frac{-8}{2(-2)}=2$ $y=f(2)=-2(2)^2+8(2)-4=4$

So, the vertex is (2 , 4)

2) Find the vertex by completing the square:

$f(x)=-2(x^2-4x)-4$

$f(x)=-2(x^2-4x+4)-4+8$

$f(x)=-2(x-2)^2+4$ *Comparing this to the form:* $f(x)=a(x-h)^2+k$ *You can see that the vertex is* $(2,4)$.

3) Graph the function with your calculator and find the vertex by using the "max" feature in the "calc" menu. (2nd trace)

Section 3.1 basically has two type of application problems:
Position Equation Problems and **Max & Min Problems**

The position equation: $h(t) = -16t^2 + v_0 t + s_0$
relates the height, *h*, of an object over time *t*, when the object is falling or projected vertically into the air.

In the position equation if the velocity is given in ft/sec then,

h = height (in feet) at time *t*
t = time (in seconds)
v_0 = initial velocity (in ft/sec)
s_0 = initial height (in feet)

EXAMPLE #1:
A ball is thrown vertically upward with an initial velocity of 48 ft/sec from a height of 10 feet.

a) What is the position equation that relates the height of the ball over time?

____

b) Sketch the graph of this function:

c) When does the ball reach its maximum height?__________

d) What is its maximum height? __________

e) How long does it take for the ball to hit the ground? __________

f) When was it 25 ft. above the ground and falling? __________

g) How high is the ball in 1 second? __________

h) When was it 50 ft. above the ground? _________

i) When is it above 40 feet? ____________________________________

There are homework assignments involving the position equation.
Problems in your textbook: Section 3.1 #77-80
and #1 and #2 on section 3.1 "additional homework handout".

The other type of application problem you will see in section 3.1 are Max & Min problems. Here's a strategy to help you solve this type of problem.

Strategy for solving Max and Min Problems in Section 3.1

1) What are you trying to maximize (or minimize)?

2) Assign a name (some letter) to this quantity that you are trying to maximize (or minimize).

3) Using the information you are given, write a formula for this quantity.

4) <u>Write your formula in terms of a single variable</u> by using other information given and substituting into the formula.

5) Your function will now probably be a quadratic function. Find the vertex by any method (unless a particular method is specified). Whether your parabola opens upward or downward should be consistent with whether you are trying to find a maximum or minimum. It's a good idea to sketch the parabola and label the axes correctly. (They probably **won't** be *x* and *y*.)

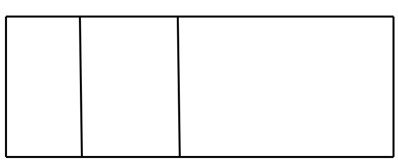
6) Your vertex is an ordered pair. The first coordinate of the ordered pair is the quantity that **produces** the maximum (or minimum) value. The second coordinate of the ordered pair **is** the maximum (or minimum).

7) Answer the questions being posed in the problem.

Examples of Max & Min problems:

Example 1: Suppose you have 1800 m of fencing with which to build 3 adjacent rectangular corrals, as shown.

a) Find the dimensions so that the total enclosed area is as large as possible.
b) What is the maximum area of the total enclosed area?



Example 2: Find two numbers adding to 20 such that the sum of their squares is as small as possible.

Example 3: What number exceeds its square by the greatest amount?

Example 4: A triangle is formed by the following procedure:

-- Draw the line $y = -\frac{2}{3}x + 5$.

-- Pick a point that is on this line and in the first quadrant. Call the coordinates of this point (*x* , *y*).

-- Draw a line from the point (*x* , *y*) to the origin and draw a vertical line from the point (*x* , *y*) to the *x*-axis.

Answer the following questions:

a) What are the coordinates of the point (*x* , *y*) that will maximize the area of the triangle formed? ________

b) What is the maximum area of the triangle formed? ________

Max & Min homework problems in your book: 3.1 #83, 85, 87

Max & Min homework problems on "additional homework handout": #3 – 7.