Worksheets solutions 10/11/19 Mrs. Robertson

MAC 1140

Section 4.2 (in-class problems)

Find the amount of time (to the nearest day) that it would take for a deposit of $5000 to grow to $12,000 at 11% compounded continuously.

$12{,}000 = 5000e^{.11t}$

$2.4 = e^{.11t}$

$\ln(2.4) = .11t$

$t = 7.958806703$

$.958806703 \times 365 = 349.9644467$

$t = 7 \text{ yrs } 350 \text{ days}$

Find the rate at which a deposit of $5000 would grow to $15,000 in 8 years, compounded continuously.

$15000 = 5000e^{8r}$

$3 = e^{8r}$

$\ln(3) = 8r$

$\frac{\ln(3)}{8} = r$

$r = .1373265361$

$r \approx 13.7\%$

102) *Deforestation in El Salvador.* It is estimated that at the present rate of deforestation in El Salvador, in 20 years only 53% of the present forest will be remaining. Use the exponential model $F = F_0 e^{rt}$ to determine the annual rate of deforestation in El Salvador.

$(F = 53\% = .53 \quad F_0 = 100\% = 1)$

$.53 = e^{20r}$

$\frac{\ln(.53)}{20} = \frac{20r}{20}$

$r = \frac{\ln(.53)}{20} = -.03174391136$

$r \approx -3.2\%$

$pH = -\log[H^+]$ where H^+ is the hydrogen ion concentration of the solution in moles per liter. Distilled water has a pH of approximately 7. A substance with a pH under 7 is called an acid and one with a pH over 7 is called a base.

What is the hydrogen ion concentration of water

$7 = -\log(H^+)$

$-7 = \log(H^+)$

$10^{-7} = H^+$

110) *Acidity in Your Stomach.* The gastric juices in your stomach have a hydrogen ion concentration of 10^{-1} moles per liter. Find the pH of your gastric juices.

$pH = -\log(10^{-1})$

$pH = -(-1)$

$\boxed{pH = 1}$

Who will have the most money at the end of 10 years? After 10 years you will have more money.

You invest $1000 at 6% per annum compounded continuously.

$A = Pe^{rt}$

$A = 1000e^{(.06 \times 10)}$

$A = \$1,822.12$

Your friend invests $800 at 7% per annum compounded semiannually.

$A = 800\left(1 + \frac{.07}{2}\right)^{(2 \cdot 10)}$

$A = 800(1.035)^{20}$

$A = \$1,591.83$

When will you each have the same amount of money?

$1000e^{.06t} = 800(1.035)^{2t}$

$e^{.06t} = .8(1.035)^{2t}$

$\ln(.8(1.035)^{2t}) = .06t$

$\ln(.8) + \ln(1.035^{2t}) = .06t$

$\ln(.8) + 2t\ln(1.035) = .06t$

$\ln(.8) = .06t - 2t\ln(1.035)$

$\ln(.8) = t(.06 - 2\ln(1.035))$

$\frac{\ln(.8)}{(.06 - 2\ln(1.035))} = t = 25.34900223$

$\boxed{t = 25 \text{ yrs } 127.4 \text{ days}}$

How long will it take for <u>your</u> money to double?

$2 = e^{.06t}$

$\frac{\ln(2)}{.06} = \frac{.06t}{.06}$

$11.55245301 = t$

$\boxed{t = 11 \text{ yrs } 201.6 \text{ days.}}$

If you had deposited $2000 instead of $1000, how long would it have taken for your money to double?

Same amount of time.

MAC 1140
Exponential Equations

Section 4.4
Additional Homework (A)

Find the EXACT solution in terms of the common or natural logarithms and then give a decimal approximation to the nearest hundredth.

1) $5^x = 8$ $\ln(5^x) = \ln(8) \Rightarrow x\ln(5) = \ln(8) \Rightarrow x = \frac{\ln(8)}{\ln(5)} \approx 1.29$

2) $e^{3x+1} = 25$ $\ln(25) = 3x+1$ $\ln(25) - 1 = 3x$ $\frac{\ln(25)-1}{3} = x \approx .74$

3) $5e^{3x} = 4$ $e^{3x} = 4/5$ $3x = \ln(4/5)$ $x = \frac{\ln(4/5)}{3} \approx -.074$

4) $2e^{-x} - 5 = 7$ $2e^{-x} = 12$ $e^{-x} = 6$ $-x = \ln(6)$ $x = -\ln(6) \approx -1.79$

5) $8 - 10^x = 6$ $2 = 10^x$ $x = \log(2) \approx .30$

6) $3^{5x+2} = 4$ $\ln(3^{5x+2}) = \ln(4)$ $(5x+2)\ln(3) = \ln(4)$ $5x+2 = \frac{\ln(4)}{\ln(3)}$ $x = \frac{1}{5}\left(\frac{\ln(4)}{\ln(3)} - 2\right)$ $x \approx -1.75$

7) $3(5^{2x}) = 20$ $5^{2x} = \frac{20}{3}$ $\ln(5^{2x}) = \ln(\frac{20}{3})$ $2x = \frac{\ln(20/3)}{\ln(5)}$ $x = .5\frac{\ln(20/3)}{\ln(5)} \approx .59$

8) $e^{2x} + e^x - 12 = 0$ $a = e^x$ $a^2 + a - 12 = 0$ $(a+4)(a-3) = 0$ $a = -4$ $e^x = -4$ $x = \ln(-4)$ ~~$a = 3$~~ $e^x = 3$ $x = \ln(3)$ $x = 1.10$

9) $5^{2x} - 3(5^x) + 2 = 0$ $a = 5^x$ $a^2 - 3a + 2 = 0$ $(a-2)(a-1) = 0$ $a = 2$ $5^x = 2 \rightarrow x = .43$ $a = 1$ $5^x = 1 \rightarrow x = 0$

10) $e^{2x} - 8e^x + 2 = 0$ $a = e^x$ $a^2 - 8a + 2 = 0$ use quad. formula $a = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(2)}}{2(1)}$

$a = \frac{8 \pm \sqrt{64-8}}{2}$

$a = \frac{8 \pm \sqrt{56}}{2} \approx 4 \pm 1.414$

$a = \frac{8 \pm 2\sqrt{14}}{2} = 4 \pm \sqrt{14}$

$e^x = 4 + \sqrt{14}$

$e^x = 4 - \sqrt{14}$

10.) $x = \ln(4+\sqrt{14}) \approx 2.05$

$x = \ln(4-\sqrt{14}) \approx -1.35$

Answers:

1) $x = \frac{\log 8}{\log 5}$ *or* $\frac{\ln 8}{\ln 5} \approx 1.292$

2) $x = \frac{\ln 25 - 1}{3} \approx 0.740$

3) $x = \frac{\ln(4/5)}{3}$ *or* $\frac{\ln 4 - \ln 5}{3} \approx -0.074$

4) $x = -\ln 6 \approx -1.792$

5) $x = \log 2 \approx 0.301$

6) $x = \frac{\frac{\log 4}{\log 3} - 2}{5} \approx -0.148$

7) $x = \frac{\log(20/3)}{2\log 5}$ *or* $\frac{\log 20 - \log 3}{2\log 5} \approx 0.589$

8) $x = \ln 3 \approx 1.099$

9) $x = \frac{\log 2}{\log 5} \approx 0.431, \quad x = 0$

10) $x = \ln\left(4 + \sqrt{14}\right) \approx 2.047$
$x = \ln\left(4 - \sqrt{14}\right) \approx -1.353$

MAC 1140
Harder Exponential & Log Equations

Section 4.4
Additional Homework (B)

Solve the following equations. When necessary, give answers rounded to the nearest thousandth.

1) $3^x = 6$
2) $4^x = 12$
3) $3^{a+2} = 5$
4) $5^{2-x} = 12$
5) $6^{1-2k} = 8$
6) $3^{2m-5} = 13$
7) $e^{k-1} = 4$
8) $e^{2-y} = 12$
9) $2e^{5a+2} = 8$
10) $10e^{3x-7} = 5$
11) $2^x = -3$
12) $(1/4)^x = -4$
13) $e^{2x} \cdot e^{5x} = e^{14}$
14) $e^{\ln x} = 3$
15) $e^{-\ln x} = 2$
16) $e^{3\ln x} = 1/8$
17) $e^{\ln x + \ln(x-2)} = 8$
18) $e^{2\ln x - \ln(x+2)} = 8/3$
19) $100(1+.02)^{3+n} = 150$
20) $500(1+.05)^{x/4} = 200$
21) $\log(t-1) = 1$
22) $\log\left(x^2\right) = 1$
23) $\log(x-3) = 1 - \log x$
24) $\log(x-6) = 2 - \log(x+15)$
25) $\ln(y+2) = \ln(y-7) + \ln 4$
26) $\ln p - \ln(p+1) = \ln 5$
27) $\ln(5+4y) - \ln(3+y) = \ln 3$
28) $\ln m + \ln(2m+5) = \ln 7$
29) $\ln(x) + 1 = \ln(x-4)$
30) $\ln(4x-2) = \ln 4 - \ln(x-2)$
31) $2\ln(x-3) = \ln(x+5) + \ln 4$
32) $\ln(k+5) + \ln(k+2) = \ln(14k)$
33) $\log_5(r+2) + \log_5(r-2) = 1$
34) $\log_4(x+3) + \log_4(x-3) = 1$
35) $\log_3(a-3) = 1 + \log_3(a+1)$
36) $\log w + \log(3w-13) = 1$
37) $\ln e^x - \ln e^3 = \ln e^5$
38) $\ln e^x - 2\ln e = \ln e^4$
39) $\log_2 \sqrt{2y^2} - 1 = 1/2$
40) $\log_2(\log_2 x) = 1$
41) $\log x = \sqrt{\log x}$
42) $\log(x^2) = (\log x)^2$

Answers:

1) 1.631
2) 1.792
3) -0.535
4) 0.456
5) -0.080
6) 3.667
7) 2.386
8) -0.485
9) -0.123
10) 2.102
11) ∅
12) ∅
13) 2
14) 3
15) 0.5
16) 0.5
17) 4
18) 4
19) 17.475
20) -75.121
21) 11
22) ±3.162
23) 5
24) 10
25) 10
26) ∅
27) 4
28) 1
29) ∅
30) 2.5
31) 11
32) 2, 5
33) 3
34) 3.606
35) ∅
36) 5
37) 8
38) 6
39) ±2
40) 4
41) 1, 10
42) 1, 100

MAC 1140 Exponential Growth Section 4.4
Additional Homework (C)

1) At 9:00 AM a Petri dish contains 200 bacteria. The doubling time for the bacteria is 1.5 hours. Answer all of the following to the nearest minute.

$400 = 200e^{1.5r}$ $\ln(2) = 1.5r$ 1.5r

$2 = e^{1.5r}$ $\frac{\ln(2)}{1.5} = r$

a) At what time will there be 2000 bacteria?

$2000 = 200e^{.4621t}$ $\ln(10) = .4621t$ $.4621 = r$

$10 = e^{.462t}$ $t = 4.982871874 \rightarrow 4\text{hrs } 59\text{min}$ 9:00 AM + 4:59 → 13:59 → 1:59 PM (a)

b) At what time were there only 20 bacteria?

$20 = 200e^{.4621t}$ $\ln(.1) = .4621t$

$.1 = e^{.4621t}$ $t = -4.982871874 \rightarrow -4\text{hrs } 59\text{min}$ 9:00 AM − 4:59 → 8:60 − 4:59 → 4:01 AM (b)

c) At what time will there be 6400 present?

$6400 = 200e^{.4621t}$ $\ln(32) = .4621t$

$32 = e^{.4621t}$ $t = 7.499969493$ $t \rightarrow 7\text{hrs } 30\text{min}$ 9:00 AM + 7:30 → 16:30 → 4:30 PM (c)

d) At what time were there only 6.25 bacteria present?

$6.25 = 200e^{.4621t}$ $.03125 = e^{.4621t}$ $t = -7.49996\ldots$

$\ln(.03125) = .4621t$ $-7\text{hrs } 30\text{min}$ 9:00 AM − 7:30 → 1:30 AM (d)

e) How many bacteria were present at 7:15 AM that morning?

7:15 to 9:00 = −1.75 hrs

$A = 200e^{.4621 \times -1.75}$ $A = 89.08957877$ A = 89 (e)

f) How many were present at 2:20 PM that afternoon?

9:00 to 2:20 = 5⅓ hrs

$A = 200e^{(.4621 \times 16/3)}$ $A = 2351.598762$ A = 2352 bacteria (f)

Answers:

a) 1:59 PM

b) 4:01 AM

c) 4:30 PM

d) 1:30 AM

e) 89 bacteria

f) 2351 bacteria