Precalculus Final Exam Review Solutions (Fall 2018)

1.) $y = 3x^2 - 6x - 24$

$y = 3(x^2 - 2x + 1) - 24 - 3$

$y = 3(x-1)^2 - 27$

$3(x-1)^2 - 27 = 0$

$3(x-1)^2 = 27$

$(x-1)^2 = 9$

$x - 1 = \pm 3$

$x = 1 \pm 3$

$\boxed{x = 4, -2}$ x-intercepts

$y = 3(0)^2 - 6(0) - 24$

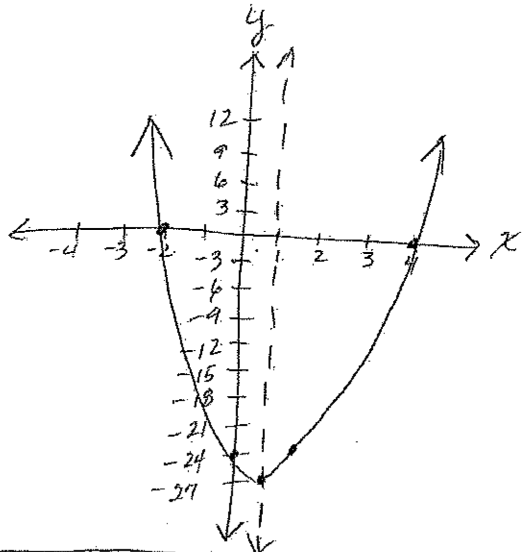
$\boxed{y = -24}$ y-intercept

$\boxed{\text{vertex: } (1, -27)}$

$\boxed{\text{axis of sym: } x = 1}$

$\boxed{\text{domain: } (-\infty, \infty)}$

$\boxed{\text{range: } [-27, \infty)}$



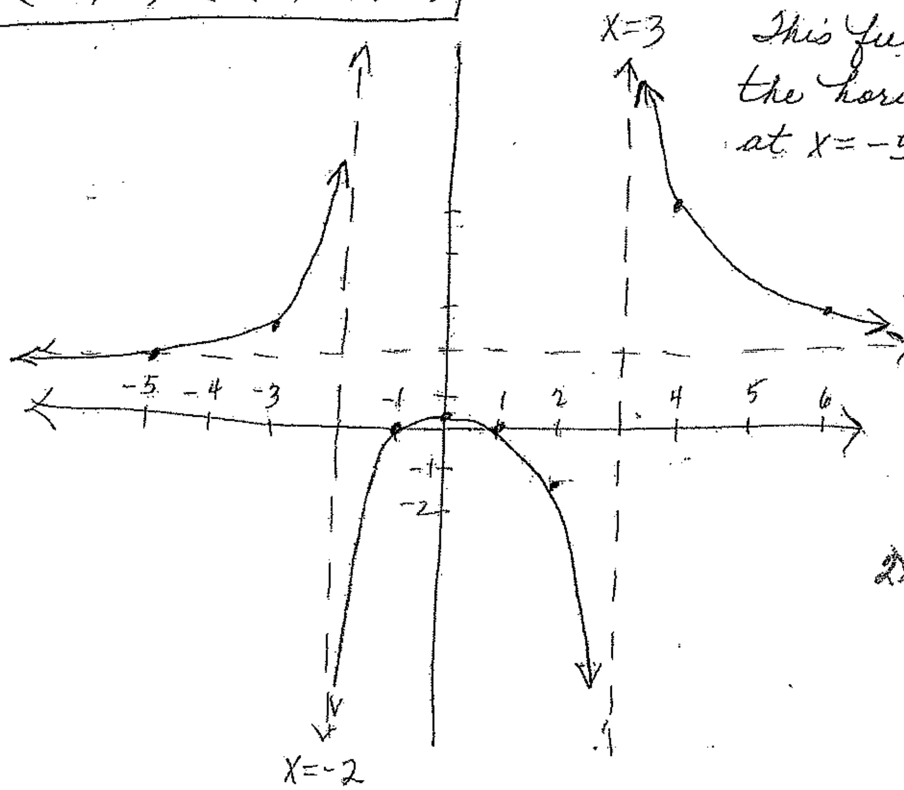
2.) $y = \dfrac{2x^2 - 2}{x^2 - x - 6} = \dfrac{2(x^2 - 1)}{(x-3)(x+2)} =$

$\boxed{\text{VA: } x = 3,\ x = -2}$ $\boxed{\text{y-intercept: } 1/3}$

$\boxed{\text{HA: } y = 2}$ $\boxed{\text{x-intercepts: } \pm 1}$

$\boxed{\text{domain: } (-\infty, -2) \cup (-2, 3) \cup (3, \infty)}$

x	y
1	0
-1	0
0	1/3
2	-3/2
4	5
6	2.9
-3	2.9
-4	2.1
-5	2



This function crosses the horizontal asymptote at $x = -5$.

$\dfrac{2x^2 - 2}{x^2 - x - 6} = \dfrac{2}{1}$

$\cancel{2x^2} - 2x - 12 = \cancel{2x^2} - 2$

$-2x - 12 = -2$

$-2x = 10$

$x = -5$

3.) We know that $f(x) = 0$ at ± 1 and $f(x) > 0$ from $(-\infty, -2) \cup (-1, 1) \cup (3, \infty)$ so

$2x^2 - 2 = 0$
$2x^2 = 2$
$x^2 = 1$
$x = \pm 1$

$$\boxed{f(x) \geq 0 \text{ at } (-\infty, -2) \cup [-1, 1] \cup (3, \infty)}$$

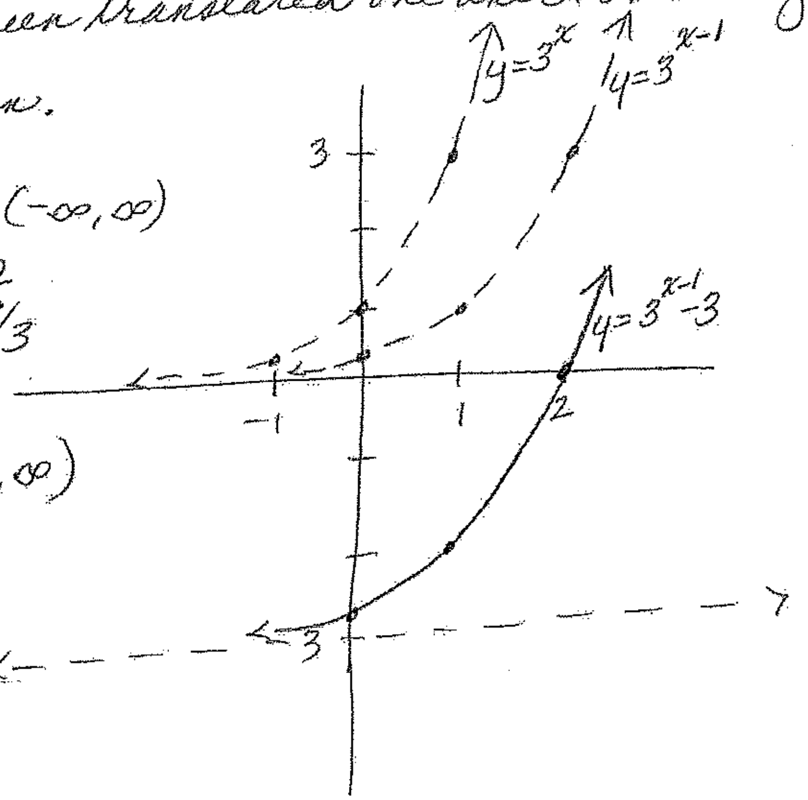
4.) $y = 3^{x-1} - 3$ This is the graph of the parent function, $y = 3^x$ that has been translated one unit to the right and 3 units down.

$y = 3^x$

x	y
-1	1/3
0	1
1	3

(parent)

domain: $(-\infty, \infty)$
x-int: 2
y-int: $-8/3$
HA: $y = -3$
range: $(-3, \infty)$



HA
$y = -3$

5.) $y = \log_2(x+4)$

exponential parent f(x)
$y = 2^x$

x	y
-1	1/2
0	1
1	2

log parent function
$y = \log_2(x)$

x	y
1/2	-1
1	0
2	1

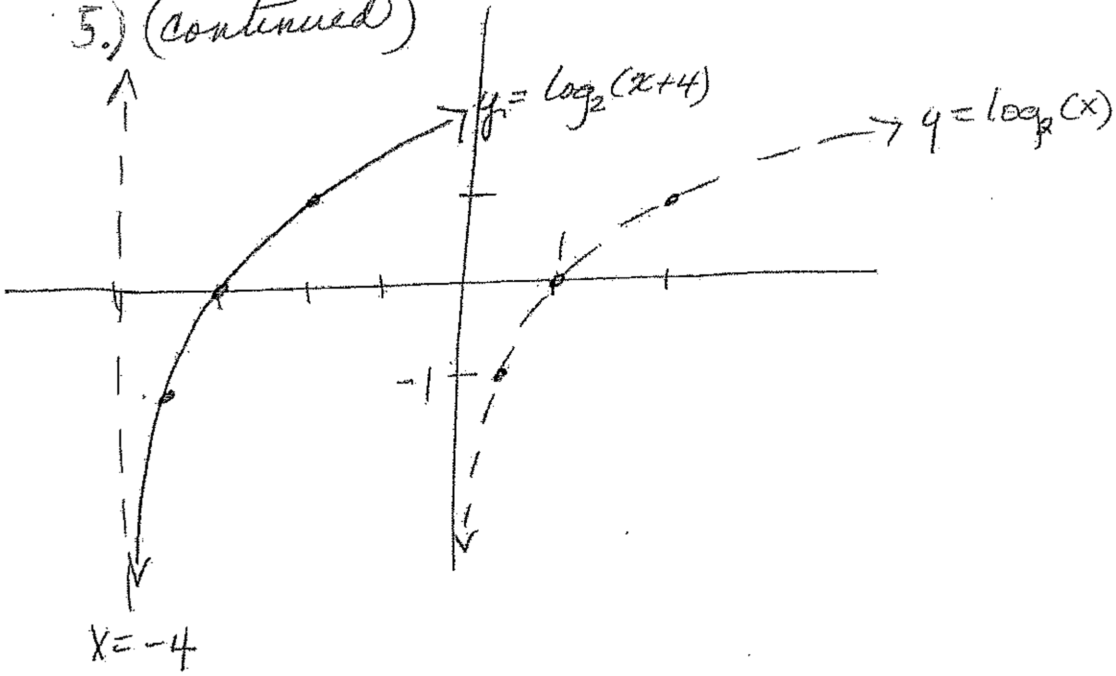
$y = \log_2(x+4)$ (moves 4 units left)

x	y
$-3\frac{1}{2}$	-1
-3	0
-2	1

(See graph on next page.)

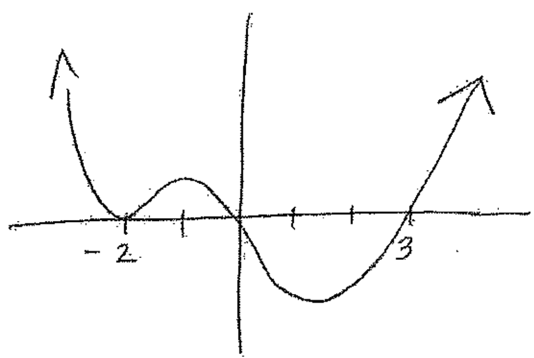
x-intercept: -3
y-intercept: 2
domain: $(-4, \infty)$
range: $(-\infty, \infty)$

5.) (continued)



6.) x-intercepts: $0, -2, 3$ (odd powers pass through - even powers "bounce")

degree is 6 $\therefore \nwarrow \nearrow$



9.) $x(x+2)^2(x-3)^3 \geq 0$ $\boxed{(-\infty, 0] \cup [3, \infty)}$ degree is odd $\therefore \swarrow \nearrow$

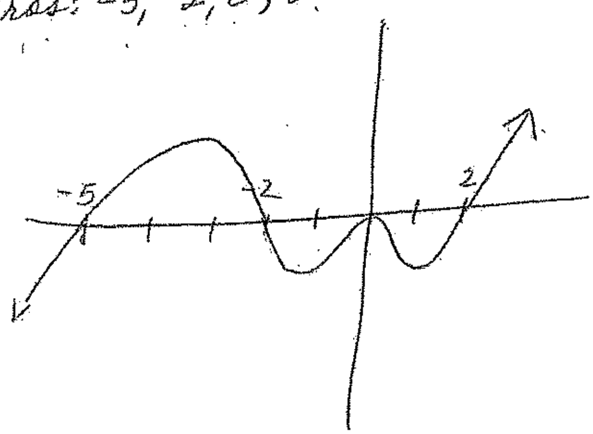
10.) $y = x^5 + 5x^4 - 4x^3 - 20x^2$ zeros: $-5, -2, 0, 2$

$y = x^2(x^3 + 5x^2 - 4x - 20)$

$x^2[x^2(x+5) - 4(x+5)]$

$y = (x^2)(x^2 - 4)(x+5)$

$y = (x^2)(x+2)(x-2)(x+5)$



11.) Evaluate: (a) $\log_3\left(\frac{1}{9}\right) = x$

$3^x = \frac{1}{9}$

$3^x = 3^{-2}$

$\boxed{x = -2}$

(b) $\ln(e^5) = \boxed{5}$ (log properties)

(c) $\log(1000) = \log 10^3 = \boxed{3}$

(d) $\ln(1) = \boxed{0}$

(e) $\log_{\frac{1}{8}}(4) = x$

$\left(\frac{1}{8}\right)^x = 4$

$(2^{-3})^x = 2^2$

$-3x = 2$

$\boxed{x = -\frac{2}{3}}$

(f) $e^{3\ln 2} = e^{\ln 2^3} = e^{\ln 8} = \boxed{8}$

(log properties)

12.) $4\ln(2x^3) - 2\ln(xy) + 3\ln(x^2y^3)$

$\ln(2x^3)^4 - \ln(xy)^2 + \ln(x^2y^3)^3$

$\ln(16x^{12}) - \ln(x^2y^2) + \ln(x^6y^9)$

$\ln\left(\frac{16x^{12}\cdot x^6y^9}{x^2y^2}\right) \rightarrow \ln\left(\frac{16x^{18}y^9}{x^2y^2}\right) \rightarrow \boxed{\ln(16x^{16}y^7)}$

13.) $x^2 - y = 5 \longrightarrow y = x^2 - 5$ Yes - y is a function of x.

domain: $(-\infty, \infty)$ range: $[-5, \infty)$

14.) $f(x) = 4x^2 - x$ $g(x) = 3x + 1$

(a) $f(a+2) = 4(a+2)^2 - (a+2)$

$= 4(a^2 + 4a + 4) - a - 2$

$= 4a^2 + 16a + 16 - a - 2$

$\boxed{f(a+2) = 4a^2 + 15a + 14}$

(b) $(f+g)(x) = f(x) + g(x)$

$= 4x^2 - x + 3x + 1$

$\boxed{(f+g)(x) = 4x^2 + 2x + 1}$

14.) (continued) $f(x) = 4x^2 - x \quad g(x) = 3x+1$

(c) $(f-g)(x) = f(x) - g(x)$

$= (4x^2 - x) - (3x+1)$

$\boxed{(f-g)(x) = 4x^2 - 4x - 1}$

$(f-g)(3) = 4(3)^2 - 4(3) - 1$

$= 36 - 12 - 1$

$\boxed{(f-g)(3) = 23}$

(d) $(f \circ g)(x) = 4(3x+1)^2 - (3x+1)$

$= 4(9x^2 + 6x + 1) - 3x - 1$

$= 36x^2 + 24x + 4 - 3x - 1$

$\boxed{(f \circ g)(x) = 36x^2 + 21x + 3}$

15.) difference quotient formula: $\dfrac{f(x+h) - f(x)}{h}$

(a) $f(x) = 3x^2 - 4$

$f(x+h) = 3(x+h)^2 - 4$

$= 3(x^2 + 2xh + h^2) - 4$

$= 3x^2 + 6xh + 3h^2 - 4$

$\dfrac{(\cancel{3x^2} + 6xh + 3h^2 \cancel{-4}) - (\cancel{3x^2} \cancel{-4})}{h}$

$\dfrac{6xh + 3h^2}{h}$

$\boxed{6x + 3h}$

(b) $g(x) = x^2 - 5x$

$g(x+h) = (x+h)^2 - 5(x+h)$

$= x^2 + 2xh + h^2 - 5x - 5h$

$\dfrac{(\cancel{x^2} + 2xh + h^2 \cancel{-5x} - 5h) - (\cancel{x^2} \cancel{-5x})}{h}$

$\dfrac{2xh + h^2 - 5h}{h}$

$\boxed{2x + h - 5}$

(c) $F(x) = \dfrac{4}{x}$

$F(x+h) = \dfrac{4}{x+h}$

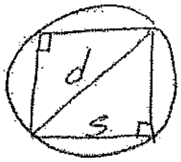
$\dfrac{\frac{4}{x+h} - \frac{4}{x}}{h} =$

$\dfrac{\frac{4(x) - 4(x+h)}{(x+h)(x)}}{h} =$

$\dfrac{\cancel{4x} \cancel{-4x} - 4h}{(x+h)(x)} \cdot \dfrac{1}{h} =$

$\dfrac{-4\cancel{h}}{(x+h)(x)\cancel{h}} =$

$\boxed{\dfrac{-4}{(x+h)(x)}}$

16.) 

(a square inscribed in a circle)

(a) $A = \pi r^2 \quad r = \frac{1}{2}d$

$A = \pi\left(\frac{1}{2}d\right)^2$

$\boxed{A = \frac{\pi d^2}{4}}$

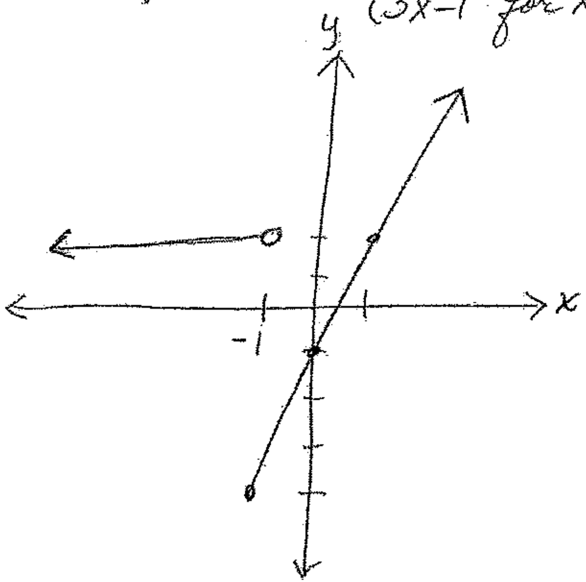
(b) $d^2 = s^2 + s^2$

$d^2 = 2s^2$

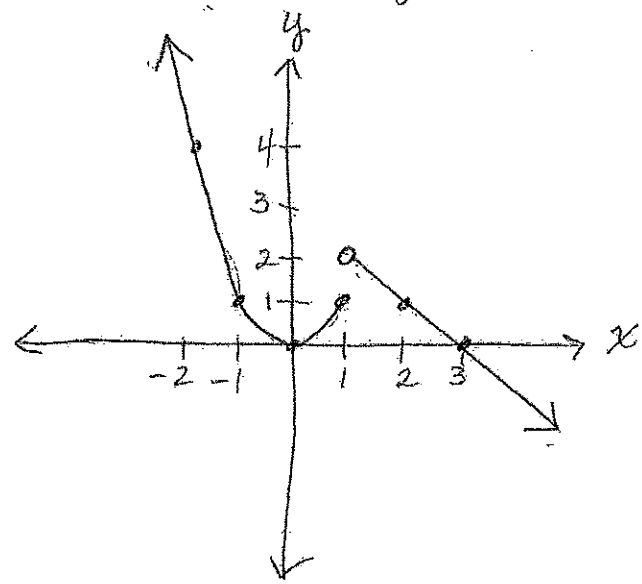
$\frac{d^2}{2} = s^2$

$\boxed{\frac{d}{\sqrt{2}} = s}$

17.) Graph $f(x) = \begin{cases} 2, & \text{for } x < -1 \\ 3x-1 & \text{for } x \geq -1 \end{cases}$



18.) $g(x) = \begin{cases} x^2, & \text{for } x \leq 1 \\ -x+3 & \text{for } x > 1 \end{cases}$



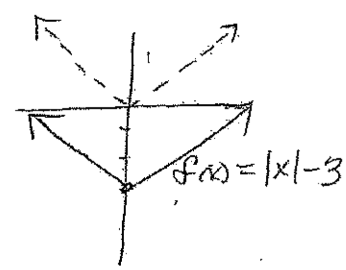
19.) $f(x) = \begin{cases} x+4 & \text{for } -\infty \leq x \leq -3 \\ -(x+2)^2+2 & -3 \leq x \leq -1 \\ 1 & -1 < x < 2 \\ x-2 & 2 \leq x \leq 3 \end{cases}$

20.) increasing: $(-\infty, -2)$, $(2, 3)$

decreasing: $(-2, -1)$

Constant: $(-1, 2)$

21.) Given: $f(x) = |x| - 3$



(a) even, because it is symmetrical about the y-axis.

(b) Not one-to-one because it fails the horizontal line test.

22.) $f = \{(3,-2),(1,-4),(5,-4)\}$ $g = \{(0,2),(-2,7),(3,1),(1,6),(-4,8)\}$

(a) $f+g = \{(3,-1),(1,2)\}$ (b) $f \circ g = \{(3,4)\}$

(c) $g^{-1} = \{(2,0),(7,-2),(1,3),(6,1),(8,-4)\}$ (d) Yes

23.) $f(x) = \dfrac{3x}{x-4}$

$$y = \frac{3x}{x-4}$$

$$x = \frac{3y}{y-4}$$

$$x(y-4) = 3y$$

$$xy - 4x = 3y$$

$$xy - 3y = 4x$$

$$y(x-3) = 4x$$

$$y = \frac{4x}{x-3}$$

$$\boxed{f^{-1}(x) = \frac{4x}{x-3}}$$

24.) $g(x) = \sqrt{x+3}$

$$y = \sqrt{x+3}$$

$$x = \sqrt{y+3}$$

$$x^2 = y+3$$

$$x^2 - 3 = y$$

$$\boxed{f^{-1}(x) = x^2 - 3}$$

25.) $x^2 \geq 4x - 1$ or

$$x^2 - 4x + 1 \geq 0$$

$$x^2 - 4x + 1 = 0$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = 2 \pm \sqrt{3}$$

$2-\sqrt{3}$ (.267) $\quad$ $2+\sqrt{3}$ (3.732)

$x = 0$	$x = 1$	$x = 4$
$0^2 \geq 4 \cdot 0 - 1$	$1^2 \geq 4 \cdot 1 - 1$	$4^2 \geq 4 \cdot 4 - 1$
$0 \geq -1$	$1 \geq 3$	$16 \geq 15$
✓	X	✓

$$\boxed{(-\infty, 2-\sqrt{3}), (2+\sqrt{3}, \infty)}$$

26.) $h(t) = -16t^2 + 100t + 5$ (for max. height use vertex formula)

$h = \dfrac{-100}{2(-16)} = 3.125$ secs. to reach max height

$\boxed{\text{max. height} = 161.25 \text{ ft}}$ $\quad h(3.125) = -16(3.125)^2 + 100(3.125) + 5 = 161.25$ ft

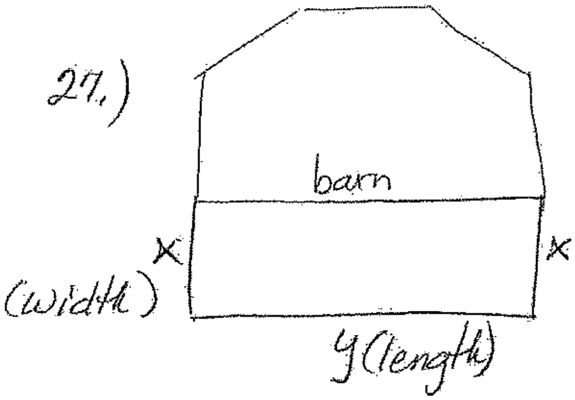
$$-16t^2 + 100t + 5 = 0 \qquad t = \frac{-100 \pm \sqrt{100^2 - 4(-16)(5)}}{2(-16)} = \frac{-100 \pm \sqrt{10320}}{-32}$$

$$t = -.0496\ldots \text{ or } 6.2996\ldots$$

X $\quad$ ✓

$\boxed{\text{The ball will hit the ground in approximately 6.2996 seconds.}}$

27.)



$50 = 2x + y \qquad A = xy$

$50 - 2x = y \qquad A = x(50-2x)$

$A = 50x - 2x^2$

The vertex (h, k) will determine the dimension of x and the maximum area.

$h = \frac{-50}{2(-2)} = 12.5$ feet $\qquad 50 = 2(12.5) + y$

$25 = y$

$\boxed{\begin{array}{l} x(\text{width}) = 12.5 \text{ feet} \\ y(\text{length}) = 25 \text{ feet} \end{array}}$

28.) $2x^4 - 13x^2 + 2x + 1 \div x + 3$

$$\begin{array}{r|rrrrr} -3 & 2 & 0 & -13 & 2 & 1 \\ & & -6 & 18 & -15 & 39 \\ \hline & 2 & -6 & 5 & -13 & 40 \end{array}$$

$\boxed{\begin{array}{l} \text{Quotient: } 2x^3 - 6x^2 + 5x - 13 \\ \text{Remainder: } 40 \end{array}}$

29.) (a) $\frac{P}{Q} = \pm \frac{2 \cdot 1}{1} = \pm\{2, 1\}$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 0 & 2 \\ \times & & 1 & 0 & 0 \\ \hline & 1 & 0 & 0 & 2 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 0 & 2 \\ & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & 0 \end{array} \qquad \begin{array}{r|rrr} 2 & 1 & -2 & 2 \\ \times & & 2 & 0 \\ \hline & 1 & 0 & 2 \end{array} \qquad \begin{array}{r|rrr} -2 & 1 & -2 & 2 \\ \times & & -2 & 8 \\ \hline & 1 & -4 & 10 \end{array}$$

$x^2 - 2x + 2 = 0 \qquad x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2}$

$x = 1 \pm i$

$\boxed{\text{Solutions: } -1, 1-i, 1+i}$

29.) (Continued) (b) $x^4 - 4x^3 - 8x^2 + 32x - 21 = 0$

$\frac{P}{Q} = \pm\left\{\frac{1, 3, 7, 21}{1}\right\} = \pm\{1, 3, 7, 21\}$

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & -8 & 32 & -21 \\ & & 1 & -3 & -11 & 21 \\ \hline & 1 & -3 & -11 & 21 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -3 & -11 & 21 \\ \times & & -1 & 4 & 7 \\ \hline & 1 & -4 & -7 & 28 \end{array} \qquad \begin{array}{r|rrrr} 3 & 1 & -3 & -11 & 21 \\ \times & & 3 & 0 & -33 \\ \hline & 1 & 0 & -11 & -12 \end{array} \qquad \begin{array}{r|rrrr} -3 & 1 & -3 & -11 & 21 \\ & & -3 & 18 & -21 \\ \hline & 1 & -6 & 7 & 0 \end{array}$$

$1 \ -6 \ 7 \rightarrow \ x^2 - 6x + 7 = 0$
not factorable

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)} = \frac{6 \pm \sqrt{36-28}}{2}$

$x = \frac{6 \pm \sqrt{8}}{2} = \frac{6 \pm 2\sqrt{2}}{2} = 3 \pm \sqrt{2}$

$\boxed{\text{solutions: } 1, -3, 3+\sqrt{2}, 3-\sqrt{2}}$

30.) $x = -2,\ x = 3+i,\ x = 3-i$

$x+2=0 \quad x-3-i=0 \quad x-3+i=0$

$(x+2)[((x-3)-i)((x-3)+i)] = 0$

$(x+2)[(x-3)^2 - i^2] = 0$

$(x+2)(x^2 - 6x + 9 + 1) = 0$

$(x+2)(x^2 - 6x + 10) = 0$

$$\begin{array}{r} x^3 - 6x^2 + 10x \\ + 2x^2 - 12x + 20 \\ \hline x^3 - 4x^2 - 2x + 20 \end{array}$$

$\boxed{P(x) = x^3 - 4x^2 - 2x + 20}$

31.) (a) $y = \frac{5x^2+3}{x^2-9}$ $\boxed{\begin{array}{l} \text{VA: } x = \pm 3 \\ \text{HA: } y = 5 \end{array}}$

(b) $y = \frac{3x+2}{x^2+1}$ $\boxed{\begin{array}{l} \text{VA: none} \\ \text{HA: } y = 0 \end{array}}$

(c) $y = \frac{2x^2 - 3x + 1}{x - 2}$ $\boxed{\begin{array}{l} \text{VA: } x = 2 \\ \text{HA: none} \\ \text{OA: } y = 2x+1 \end{array}}$

$$\begin{array}{r|rrr} 2 & 2 & -3 & 1 \\ & & 4 & 2 \\ \hline & 2 & 1 & \cancel{3} \end{array}$$ omit remainder

$y = 2x + 1$

32.) VA: $x=2 \rightarrow$ means $(x-2)$ in denominator
HA: $y=3 \rightarrow 3x+b$ in numerator

So $y=\dfrac{3x+b}{x-2}$ substitute the x-intercept

$0=3(4)+b$
$0=12+b$
$-b=12$
$b=-12 \rightarrow$ numerator is $3x-12$

$$\boxed{y=\frac{3x-12}{x-2}}$$

33.) $5(e^x)-3=4$

(a) $5e^x=7$
$e^x=7/5$
$\boxed{x=\ln(7/5) \quad x\approx .34}$

(b) $\sqrt{3}\cdot 3^{x+2}=9$
$\underbrace{3^{1/2}\cdot 3^{x+2}}=3^2$
$3^{x+5/2}=3^2$
$x+5/2=2$
$x=2-5/2$
$\boxed{x=-1/2}$

(c) $\log_3(x^2-5)=2$
$x^2-5=3^2$
$x^2-5=9$
$x^2=14$
$\boxed{x=\pm\sqrt{14} \quad x=\pm 3.74}$

(d) $\log x=1+\log(x-3)$
$\log(x)-\log(x-3)=1$
$\log\left(\frac{x}{x-3}\right)=1$
$\frac{x}{x-3}=10$
$x=10(x-3)$
$x=10x-30$
$-9x=-30$
$\boxed{x=10/3}$

(e) $e^{2x}-3e^x-10=0$
(Let $u=e^x$)
$u^2-3u-10=0$
$(u-5)(u+2)=0$
$u=5 \quad u=-2$
(back substitute)
$e^x=5$
$\boxed{x=\ln(5) \quad x\approx 1.61}$

$e^x=-2$
$\cancel{x=\ln(-2)}$

33.) (continued)

(f) $(\ln(x))^2 + 2\ln(x) - 8 = 0$

(Let $u = \ln(x)$)

$u^2 + 2u - 8 = 0$

$(u+4)(u-2) = 0$

$u = -4 \quad u = 2$

(back substitute)

$\ln x = -4 \quad \ln x = 2$

$$\boxed{\begin{array}{ll} e^{-4} = x & e^2 = x \\ x \approx .02 & x \approx 7.39 \end{array}}$$

(g) $\log_4(x) + \log_4(2x+5) = \log_4(3)$

$\log_4(x(2x+5)) = \log_4(3)$

$x(2x+5) = 3$

$2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$

$2x = 1 \quad x = -3$

$x = 1/2 \quad \times$

34.) $A = Pe^{rt}$

$8000 = Pe^{.04 \times 5}$

$\frac{8000}{e^{.04 \times 5}} = P$

$\boxed{\$6549.85 \approx P}$

35.) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 2500\left(1 + \frac{.032}{12}\right)^{(12 \cdot 8)}$

$\boxed{A = \$3228.28}$

36.) $2.5 = 5e^{32r}$

$.5 = e^{32r}$

$\ln(.5) = 32r$

$\frac{\ln(.5)}{32} = r$

$r \approx -.02166$

(rate of decay)

$A = 5e^{-.02166 \times 41}$

$A = 2.057\ldots$

$\boxed{A \approx 2.1 \text{ grams}}$

$1 = 5e^{-.02166t}$

$.2 = e^{-.02166t}$

$\ln(.2) = -.02166t$

$\frac{\ln(.2)}{-.02166} = t$

$\boxed{\begin{array}{l} 74.30461276 = t \text{ (years)} \\ t \approx 74 \text{ yrs } 3 \text{ mos } 20 \text{ days} \end{array}}$

37.) $500 = 210e^{3r}$

$\frac{500}{210} = e^{3r}$

$\ln\left(\frac{50}{21}\right) = 3r$

$\frac{\ln\left(\frac{50}{21}\right)}{3} = r$

$r = .289167$

(rate of growth)

$750 = 210e^{.289167t}$

$\frac{75}{21} = e^{.289167t}$

$\ln\left(\frac{75}{21}\right) = .289167t$

$\frac{\ln\left(\frac{75}{21}\right)}{.289167} = t$

$t = 4.402181701$ hours

$t = 4$ hrs 24 minutes

8:00 am
+ 4:24

$\boxed{12:24\text{pm}}$

At 12:24 pm there will be 750 bacteria

$A = 210e^{.289167 \times 5}$

$A = 210e^{1.445835}$

$A = 891.5330694$

$\boxed{\text{At 1100 pm there are} \approx 892 \text{ bacteria}}$

38.) (a) $y = 3x^2$
$y = 2x + 5$

$3x^2 = 2x + 5$

$3x^2 - 2x - 5 = 0$

$(3x - 5)(x + 1) = 0$

$x = 5/3, -1$

$y = 2(5/3) + 5$

$y = 10/3 + 15/3$

$y = \frac{25}{3}$

$\boxed{\left(5/3, 25/3\right)}$

$y = 2(-1) + 5$

$y = 3$

$\boxed{(-1, 3)}$

(b) $y = 9^{x-1}$
$y = 3^{4x+3}$

$9^{x-1} = 3^{4x+3}$

$(3^2)^{x-1} = 3^{4x+3}$

$3^{2x-2} = 3^{4x+3}$

$2x - 2 = 4x + 3$

$-5 = 2x$

$-5/2 = x$

$y = 3^{4(-5/2)+3}$

$y = 3^{-10+3}$ or 3^{-7}

$y = \frac{1}{2187}$

$\boxed{\left(-5/2, \frac{1}{2187}\right)}$

(c) $x^2 - 3y^2 = 13$
$x - 3y = 1 \rightarrow x = 3y + 1$

$(3y+1)^2 - 3y^2 = 13$

$9y^2 + 6y + 1 - 3y^2 = 13$

$6y^2 + 6y - 12 = 0$

$y^2 + y - 2 = 0$

$(y + 2)(y - 1) = 0$

$y = 2, -1$

$x = 3(2) + 1$

$x = 7$

$x = 3(-1) + 1$

$x = -2$

$\boxed{(7, 2) \text{ \& } (-2, -1)}$

39.) (a) $x^2-8x+y^2+14y+40=0$

$(x^2-8x+\underline{16})+(y^2+14y+\underline{49})=-40+\underline{16}+\underline{49}$

$(x-4)^2+(y+7)^2=25$

circle with center $(4,-7)$
and radius of 5

(b) $2x^2+12x-y+14=0$

$2x^2+12x=y-14$

$2(x^2+6x+\underline{9})=y-14+\underline{18}$

$2(x+3)^2=y+4$

$2(x+3)^2-4=y$
parabola with vertex $(-3,-4)$
opens up.

40.) $\dfrac{(x-1)^2}{4}-\dfrac{(y+2)^2}{16}=1$

$c^2=4+16$
$c^2=20$
$c=\pm 2\sqrt{5}$

$m=\pm\frac{1}{2}$

$y+2=\frac{1}{2}(x-1)$
$y=\frac{1}{2}x-\frac{1}{2}-2$
$y=\frac{1}{2}x-\frac{5}{2}$

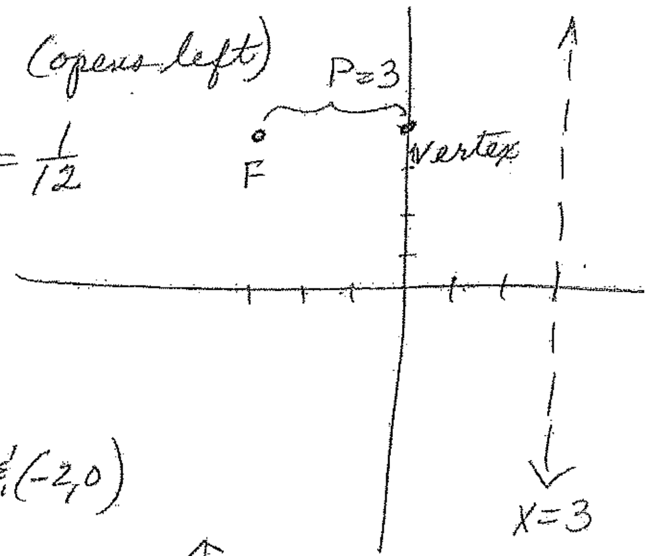
$y+2=-\frac{1}{2}(x-1)$
$y=-\frac{1}{2}x+\frac{1}{2}-2$
$y=-\frac{1}{2}x-\frac{3}{2}$

center: $(1,-2)$
foci: $(1+2\sqrt{5},-2)$ $(1-2\sqrt{5},-2)$
asymptotes: $y=\frac{1}{2}x-\frac{5}{2}$
$y=-\frac{1}{2}x-\frac{3}{2}$

41.) focus $(-3,4)$ directrix: $x=3$ (opens left)

vertex $(0,4)$ $\quad a = \frac{1}{4p} = \frac{1}{4\cdot 3} = \frac{1}{12}$

$$\boxed{x = -\tfrac{1}{12}(y-4)^2}$$



42.) vertices $(-2,5)$ & $(-2,1)$ foci $(-2,6)$ & $(-2,0)$

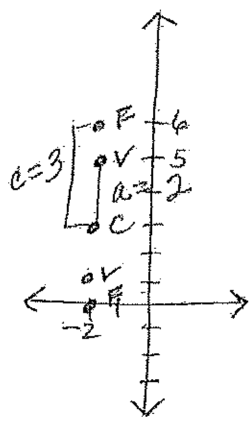
center $(-2,3)$ $\quad a=2 \quad c=3$

$c^2 = a^2 + b^2 \quad a^2 = 4 \quad c^2 = 9$

$3^2 = 2^2 + b^2$

$5 = b^2$

$$\boxed{\frac{(y-3)^2}{4} - \frac{(x+2)^2}{5} = 1}$$



43.) $3\begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} - x\begin{vmatrix} -2 & -5 \\ 1 & 1 \end{vmatrix} + -6\begin{vmatrix} -2 & -5 \\ 3 & 4 \end{vmatrix} = 3$

$3(3\cdot 1 - 4\cdot 1) - x(-2\cdot 1 - 1\cdot{}^-5) - 6(-2\cdot 4 - 3\cdot{}^-5) = 3$

$3(-1) - x(3) - 6(7) = 3$

$-3 - 3x - 42 = 3$

$-45 - 3 = 3x$

$-48 = 3x$

$\boxed{-16 = x}$

46.)

$$\begin{array}{cccc} (x) & (y) & (z) & \\ 2 & -3 & 1 & 5 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{ccc} a & b & c \end{array}$$
$$D = \begin{vmatrix} 2 & -3 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{vmatrix}$$

$D_x = \begin{vmatrix} 5 & -3 & 1 \\ 2 & 1 & -3 \\ 0 & 0 & 0 \end{vmatrix} \quad D_y = \begin{vmatrix} 2 & 5 & 1 \\ 0 & 2 & -3 \\ 0 & 0 & 0 \end{vmatrix} \quad D_z = \begin{vmatrix} 2 & -3 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{vmatrix}$

$\boxed{\text{44.) Omit}}$

$D = \begin{vmatrix} 2 & -3 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{vmatrix} = 2\begin{vmatrix} +1 & -3 \\ 0 & 0 \end{vmatrix} + 0 + 0$

$= 2(0-0) + 0 + 0$

$= 0$

Since $D=0$ Cramer's Rule cannot be used to solve the system

We can eliminate the ys or we can eliminate the zs but we can't eliminate the xs.

Since we really only have 2 equations involving 3 variables — this is a dependent system $\longrightarrow$

46.) (continued) x(d b c) y(a d c) z(a b d)

(1) $2x+3y+z=5$ $\quad -(3)(y-3z=2)$ $\quad 2x+3y+z=5$

(2) $0x+y-3z=2$ $\quad -3y+9z=-6$ $\quad \underline{-3y+9z=-6}$

(3) $0x+0y+0z=0$ $\quad 2x+10z=-1$

$2x=-10z-1$

$y=3z+2$ $\quad x=-5z-\frac{1}{2}$

$(-5z-\frac{1}{2},\ 3z+2,\ z)$

45.) Solve #44 Using Cramer's Rule for 3X3 system

$$\begin{cases} x+2y-3z=4 \\ 2x-4y+5z=-3 \\ 5x-6y+4z=-7 \end{cases}$$

$D=\begin{vmatrix}1&2&-3\\2&-4&5\\5&-6&4\end{vmatrix}$ $\quad D_x=\begin{vmatrix}4&2&-3\\-3&-4&5\\-7&-6&4\end{vmatrix}$ $\quad x=\frac{D_x}{D}$

$y=\frac{D_y}{D}$

$D_y=\begin{vmatrix}1&4&-3\\2&-3&5\\5&-7&4\end{vmatrix}$ $\quad D_z=\begin{vmatrix}1&2&4\\2&-4&-3\\5&-6&-7\end{vmatrix}$ $\quad z=\frac{D_z}{D}$

$D=1\begin{vmatrix}-4&5\\-6&4\end{vmatrix}-2\begin{vmatrix}2&-3\\-6&4\end{vmatrix}+5\begin{vmatrix}2&-3\\-4&5\end{vmatrix}$ $\quad x=\frac{8}{24}=\frac{1}{3}$

$1[(-4\cdot4)-(-6\cdot5)]-2[(2\cdot4)-(-6\cdot-3)]+5[(2\cdot5)-(-4\cdot-3)]$ $\quad y=\frac{88}{24}=\frac{11}{3}$

$(-16+30)-2(8-18)+5(10-12)$ $\quad z=\frac{10}{24}=\frac{5}{12}$

$14+20-10$

$\underline{D=24}$

$D_x=4\begin{vmatrix}-4&5\\-6&4\end{vmatrix}+3\begin{vmatrix}2&-3\\-6&4\end{vmatrix}+7\begin{vmatrix}2&-3\\-4&5\end{vmatrix}=4[(-16+30)]+3[(8-18)]+7[(10-12)]$

$=4(14)+3(-10)+7(-2)$

$=52-30-14=8$ $\quad \underline{D_x=8}$

$D_y=1\begin{vmatrix}-3&5\\-7&4\end{vmatrix}-2\begin{vmatrix}4&-3\\-7&4\end{vmatrix}+5\begin{vmatrix}4&-3\\-3&5\end{vmatrix}=1(-12+35)-2(16-21)+5(20-9)$

$=23-2(-5)+5(11)=88$ $\quad \underline{D_y=88}$

$D_z=1\begin{vmatrix}-4&-3\\-6&-7\end{vmatrix}-2\begin{vmatrix}2&4\\-6&-7\end{vmatrix}+5\begin{vmatrix}2&4\\-4&-3\end{vmatrix}=1(28-18)-2(-14+24)+5(-12+16)$

$10-20+20=10$ $\quad \underline{D_z=10}$

$\boxed{(\frac{1}{3},\ \frac{11}{3},\ \frac{5}{12})}$

47.) $\dfrac{5x^2+2}{x(x+3)^2(x^2+5)} = \boxed{\dfrac{A}{x} + \dfrac{B}{x+3} + \dfrac{C}{(x+3)^2} + \dfrac{Dx+E}{x^2+5}}$

48) $\dfrac{10x^2-9x+3}{\cancel{x^3-5x^2+x-5}} = \dfrac{10x^2-9x+3}{(x^2+1)(x-5)} = \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x-5}$

$x^2(x-5)+1(x-5)$

$10x^2-9x+3 = (Ax+B)(x-5) + (x^2+1)(C)$

$10x^2-9x+3 = Ax^2+Bx-5Ax-5B+Cx^2+C$

$10x^2 = Ax^2+Cx^2 \longrightarrow$ $\left[\begin{array}{l} 10 = A+C \\ -9 = -5A+B \\ 3 = -5B+C \end{array}\right]$

$-9x = -5Ax+Bx \longrightarrow$

$$\begin{array}{r} 10 = A+C \\ -3 = 5B-C \\ \hline 7 = A+5B \end{array}$$

$$\begin{array}{r} +35 = 5A+25B \\ -9 = -5A+B \\ \hline 26 = 26B \end{array}$$

$\boxed{1 = B}$

$10 = 2+C$

$\boxed{8 = C}$

$-9 = -5A+1$

$-10 = -5A$

$\boxed{2 = A}$

$$\boxed{\frac{10x^2-9x+3}{(x^2+1)(x-5)} = \frac{2x+1}{x^2+1} + \frac{8}{x-5}}$$

49.) $8, 6, 4, 2, 0, -2, \ldots$ $(n \geq 1)$

(a) $8-2n$, where n is the counting numbers (natural numbers)

(b) $a_n = a_{n-1} - 2$

(c) Arithmetic

50.) $a_n = \dfrac{(n+3)!}{2n}$ $\quad a_1 = \dfrac{(1+3)!}{2(1)} = \dfrac{4!}{2} = 12$ $\quad a_2 = \dfrac{(2+3)!}{2(2)} = \dfrac{5!}{4} = 30$

$a_3 = \dfrac{(3+3)!}{2(3)} = \dfrac{6!}{6} = 120$ $\quad a_4 = \dfrac{(4+3)!}{2(4)} = \dfrac{7!}{8} = 630$

(a) $\boxed{12, 30, 120, 630, \ldots}$ $\quad$ $\boxed{\text{(b) neither}}$

51.) $r = \frac{1}{2}$ $a_2 = 6$ $a_n = 12\left(\frac{1}{2}\right)^{n-1}$

$a_1 = 12$ a_2 $\frac{1}{2} \cdot 12 = 6$ $a_3 = \left(\frac{1}{2} \cdot \frac{1}{2}\right)12 = 3$

$a_{15} = 12\left(\frac{1}{2}\right)^{14} = 12\left(\frac{1}{2^{14}}\right) = \boxed{\frac{3}{4096}}$

52.) If $a_1 = \frac{2}{3}$ and $a_5 = 0$, what is a_8? $a_n = a_1 + (n-1)d$

$a_5 = a_1 + (5-1)d$ $0 = \frac{2}{3} + (5-1)d$ $a_8 = a_1 + (n-1)\left(-\frac{1}{6}\right)$

$-\frac{2}{3} = 4d$ $a_8 = \frac{2}{3} + (8-1)\left(-\frac{1}{6}\right)$

$-\frac{1}{6} = d$ $a_8 = \frac{2}{3} - \frac{7}{6}$

$\boxed{a_8 = -\frac{1}{2}}$

53.) Write summation notation with index i and $i=1$

$\boxed{\sum_{i=1}^{5} \frac{(-1)^n}{2n+1}}$

54.) $a_1 = \sqrt{2}$ $a_2 = \sqrt{3} \cdot \sqrt{2}$ $a_3 = \sqrt{3} \cdot \sqrt{6}$ $a_4 = \sqrt{3} \cdot 3\sqrt{2}$

(a.) $a_2 = \sqrt{6}$ $a_3 = \sqrt{18}$ or $3\sqrt{2}$ $a_4 = 3\sqrt{6}$ $(\sqrt{54} = 3\sqrt{6})$

$\boxed{\sqrt{2}, \sqrt{6}, \sqrt{18}, \sqrt{54} \text{ or } \sqrt{2}, \sqrt{6}, 3\sqrt{2}, 3\sqrt{6}}$ (b) Geometric

55.) $S = \frac{56}{1 - \frac{3}{4}} = \frac{56}{\frac{1}{4}} = \boxed{224 \text{ inches}}$

56.) (a) $\sum_{i=2}^{10} 2^{i-2}$ $a = 1$ $r = 2$ $n = 9$ $S_9 = \frac{1(1-2^9)}{1-2} = \frac{1-512}{-1} = \boxed{511}$

(b) $210 + 105 + 52.5 + 26.25 + \ldots$ $a = 210$ $r = \frac{1}{2}$ $S = \frac{210}{1-\frac{1}{2}} = \boxed{420}$

(c) $a_1 = 310$ $d = -2$ $a_n = a_1 + (n-1)d$ $S_n = \frac{n}{2}(a_1 + a_n)$

$220 = 310 + (n-1)(-2)$ $S_{46} = \frac{46}{2}(310 + 220)$

$-90 = -2n + 2$ $S_{46} = 23(530) = 12{,}190$

$-92 = -2n$

$46 = n$

57) $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$, where $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Write the first four terms of $(x^2+3y)^7$

$$\binom{7}{0}(x^2)^7(3y)^0 + \binom{7}{1}(x^2)^6(3y)^1 + \binom{7}{2}(x^2)^5(3y)^2 + \binom{7}{3}(x^2)^4(3y)^3$$

$$x^{14} + 7(x^{12})(3y) + 21(x^{10})(9y^2) + 35(x)^8(27y^3)$$

$$\boxed{x^{14} + 21x^{12}y + 189x^{10}y^2 + 945x^8y^3}$$