Practice test 11/14/19

College Precalculus MAC 1140 Chapters 9 & 11 Name: Solutions / Corrections

SHOW WORK ON ALL PROBLEMS!

1. Write the first three terms of the binomial notation for $(2x - y)^{13}$.

$\sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$ $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ ($_nC_r$)

$\sum_{r=0}^{13} \binom{13}{0}(2x)^{13}(-y)^0 = 1(2x)^{13}(1) + (13)(2x)^{12}(-y) + (78)(2x)^{11}(-y)^2$

$= 8192x^{13} - 53{,}248x^{12}y + 159{,}744x^{11}y^2 + \cdots$

2. Write the binomial expansion for $(x^2 + 2y)^4$.

$\sum_{r=0}^{4} \binom{4}{0}(x^2)^4(-y)^0 = 1(x^2)^4(1) + 4(x^2)^3(2y) + 6(x^2)^2(2y)^2 + 4(x^2)(2y)^3 + 1(2y)^4$

$x^8 + 8x^6y + 24x^4y^2 + 32x^2y^3 + 16y^4$

Write a formula for the ***nth*** term of each infinite sequence. Do not use a recursion formula.

3. 3, -5, 7, -9, ⋯

$a_n = (-1)^{n+1}(2n+1)$

4. 21, 18, 15, 12, ⋯

$a_n = 21 + (n-1)(-3)$

$a_n = 24 - 3n$

Charles' parents deposited $1,000 on his 1st birthday. They deposited money in this account each year on his birthday increasing the amount by $200 each year.

5. Write in summation notation the formula depicting the amount Charles' parents deposited from his 1st birthday to his 18th birthday.

$\sum_{n=1}^{18} [1000 + 200(n-1)]$

6. What is the total amount of money Charles' parents deposited?

$\sum_{n=1}^{18} [1000 + 200(n-1)] = 48{,}600$ OR if your calculator doesn't have Σ notation

$\underset{(1)}{1000} + \underset{(2)}{1200} + \underset{(3)}{1400} + \underset{(4)}{1600} + \underset{(5)}{1800} + \underset{(6)}{2000} + \underset{(7)}{2200} + \underset{(8)}{2400} + \underset{(9)}{2600} + \underset{(10)}{2800} + \underset{(11)}{3000} + \underset{(12)}{3200} + \underset{(13)}{3400} +$

$\underset{(14)}{3600} + \underset{(15)}{3800} + \underset{(16)}{4000} + \underset{(17)}{4200} + \underset{(18)}{4400} = \$48{,}600$

7. Solve using Cramer's Rule.

$2x - y + z = 9$
$x + y + z = 4$
$-3x - 2y - z = -13$

$\begin{matrix} 2 & -1 & 1 & 9 \\ 1 & 1 & 1 & 4 \\ -3 & -2 & -1 & -13 \end{matrix}$

8. Solve using Gaussian Elimination.

$2x + 5y = 2$
$-x - 3y = -2$

See next pages for solutions to 7. and 8.

7.)

(x)	(y)	(z)	#
2	-1	1	9
1	1	1	4
-3	-2	-1	-13

$$D = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 1 & 1 \\ -3 & -2 & -1 \end{vmatrix} = 2\begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} - (-1)\begin{vmatrix} -1 & 1 \\ -2 & -1 \end{vmatrix} + (-3)\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$2(-1+2) - 1(1+2) - 3(-1-1)$$

$$2 - 3 + 6 = 5$$

$$D_x = \begin{vmatrix} 9 & -1 & 1 \\ 4 & 1 & 1 \\ -13 & -2 & -1 \end{vmatrix} = 9\begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} - 4\begin{vmatrix} -1 & 1 \\ -2 & -1 \end{vmatrix} + (-13)\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$9(-1+2) - 4(1+2) - 13(-1-1)$$

$$9 - 12 + 26 = 23$$

$$D_y = \begin{vmatrix} 2 & 9 & 1 \\ 1 & 4 & 1 \\ -3 & -13 & -1 \end{vmatrix} = 2\begin{vmatrix} 4 & 1 \\ -13 & -1 \end{vmatrix} - 1\begin{vmatrix} 9 & 1 \\ -13 & -1 \end{vmatrix} + (-3)\begin{vmatrix} 9 & 1 \\ 4 & 1 \end{vmatrix}$$

$$2(-4+13) - 1(-9+13) - 3(9-4)$$

$$18 - 4 - 15 = -1$$

$$D_z = \begin{vmatrix} 2 & -1 & 9 \\ 1 & 1 & 4 \\ -3 & -2 & -13 \end{vmatrix} = 2\begin{vmatrix} 1 & 4 \\ -2 & -13 \end{vmatrix} - 1\begin{vmatrix} -1 & 9 \\ -2 & -13 \end{vmatrix} + (-3)\begin{vmatrix} -1 & 9 \\ 1 & 4 \end{vmatrix}$$

$$= 2(-13+8) - 1(13+18) - 3(-4-9)$$

$$-10 - 31 + 39 = -2$$

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad z = \frac{D_z}{D}$$

$$x = \frac{23}{5} \qquad y = \frac{-1}{5} \qquad z = \frac{-2}{5}$$

$$\left(\frac{23}{5}, -\frac{1}{5}, -\frac{2}{5}\right)$$

Check for #7

$2\left(\frac{23}{5}\right) - \left(\frac{-1}{5}\right) + \left(\frac{-2}{5}\right) = 9$ ✓

$\frac{23}{5} - \frac{1}{5} - \frac{2}{2} = 4$ ✓

$-3\left(\frac{23}{5}\right) - 2\left(\frac{-1}{5}\right) - \left(\frac{-2}{5}\right) = -13$ ✓

8.) $2x + 5y = 2$
$-x - 3y = -2$

$$\begin{matrix} 2 & 5 & 2 \\ -1 & -3 & -2 \end{matrix}$$

$$\begin{matrix} 1 & 2 & 0 \\ -1 & -3 & -2 \end{matrix} \quad (R_2 + R_1 \to R_1)$$

$$\begin{matrix} 1 & 2 & 0 \\ 0 & -1 & -2 \end{matrix} \quad (R_2 + R_1 \to R_2)$$

$$\begin{matrix} 1 & 2 & 0 \\ 0 & 1 & 2 \end{matrix} \quad (-1 \cdot R_2 \to R_2)$$

$$\begin{matrix} 1 & 0 & -4 \\ 0 & 1 & 2 \end{matrix} \quad (-2 \cdot R_2 + R_1 \to R_1)$$

$$\begin{matrix} -2(0 & 1 & 2) \\ 0 & -2 & -4 \\ 1 & 2 & 0 \\ \hline 1 & 0 & -4 \end{matrix}$$

$x = -4 \qquad y = 2$

$$\boxed{(-4, 2)}$$

Check:

$2(-4) + 5(2) \stackrel{?}{=} 2$

$-8 + 10 = 2$ ✓

$-(-4) - 3(2) \stackrel{?}{=} -2$

$4 - 6 = -2$ ✓