College Trigonometry 6.3, 6.4 and 6.6 Practice Test Solutions

1.) $\sin\alpha = -\frac{12}{13}$ $\quad \cos\beta = \frac{1}{6}$ $\quad \tan(\alpha) = \frac{\sin\alpha}{\cos\alpha}$ $\quad \tan\beta = \frac{\sin\beta}{\cos\beta}$

$\left(-\frac{12}{13}\right)^2 + \cos^2\alpha = 1$ $\quad \sin^2\beta + \left(\frac{1}{6}\right)^2 = 1$ $\quad \tan\alpha = \frac{-\frac{12}{13}}{\frac{5}{13}}$ $\quad \tan\beta = \frac{\frac{\sqrt{35}}{6}}{\frac{1}{6}}$

$\cos^2\alpha = 1 - \frac{144}{169}$ $\quad \sin^2\beta = 1 - \frac{1}{36}$

$\cos^2\alpha = \frac{25}{169}$ $\quad \sin^2\beta = \frac{35}{36}$

$\boxed{\cos\alpha = \frac{5}{13}}$ $\quad \boxed{\sin\beta = \frac{\sqrt{35}}{6}}$ $\quad \boxed{\tan\alpha = -\frac{12}{5}}$ $\quad \boxed{\tan\beta = \sqrt{35}}$

(a) $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$

$= \frac{-\frac{12}{5} + \sqrt{35}}{1 - \left(-\frac{12}{5}\right)\left(\sqrt{35}\right)}$

$= \frac{\frac{-12 + 5\sqrt{35}}{5}}{\frac{5 + 12\sqrt{35}}{5}}$

$\boxed{\tan(\alpha+\beta) = \frac{-12 + 5\sqrt{35}}{5 + 12\sqrt{35}}}$

(b) $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$

$= 2\left(-\frac{12}{13}\right)\left(\frac{5}{13}\right)$

$\boxed{\sin(2\alpha) = -\frac{120}{169}}$

(c) $\cos\left(\frac{1}{2}\beta\right) = \pm\sqrt{\frac{1+\cos\beta}{2}}$

$= \sqrt{\frac{1 + \frac{1}{6}}{2}}$

$= \sqrt{\frac{7}{6} \cdot \frac{1}{2}}$

$= \sqrt{\frac{7}{12} \cdot \frac{12}{12}}$

$= \sqrt{\frac{84}{144}}$

$\boxed{\cos\left(\frac{1}{2}\beta\right) = \frac{2\sqrt{21}}{12}}$

(d) $\tan(3\alpha) = \tan(2\alpha + \alpha)$

$= \frac{\tan(2\alpha) + \tan(\alpha)}{1 - \tan(2\alpha)\tan(\alpha)}$

$= \frac{\frac{120}{119} + \frac{-12}{5}}{1 - \left(\frac{120}{119}\right)\left(\frac{-12}{5}\right)}$

$= \frac{\frac{-828}{595}}{\frac{407}{119}}$

$= \boxed{-\frac{828}{2035}}$

$\tan(2\alpha) = \frac{2\tan(\alpha)}{1 - \tan^2\alpha}$

$= \frac{2\left(-12/5\right)}{1 - \left(-12/5\right)^2}$

$= \frac{-\frac{24}{5}}{1 - \frac{144}{25}}$

$= \frac{-\frac{24}{5}}{\frac{-119}{25}}$

$= -\frac{24}{5} \cdot \frac{25^5}{-119}$

$\tan(2\alpha) = \frac{120}{119}$

2.) (a) $\sin(165^\circ) = \sin(120^\circ + 45^\circ)$

$= \sin(120^\circ)\cos(45) + \cos(120^\circ)\sin(45^\circ)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \text{ or } \frac{\sqrt{6}-\sqrt{2}}{4}}$

(b) $\cos(-15^\circ) = \cos(30 - 45^\circ)$

$= \cos(30)\cos(45) + \sin(30)\sin(45)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \text{ or } \frac{\sqrt{6}+\sqrt{2}}{4}}$

(c) $\tan(75^\circ) = \tan(45^\circ + 30^\circ)$ $\left(\tan(45^\circ) = 1 \text{ & } \tan(30^\circ) = \frac{\sqrt{3}}{3}\right)$

$= \frac{\tan(45) + \tan(30)}{1 - \tan(45)\tan(30)}$

$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - (1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \boxed{\frac{3+\sqrt{3}}{3-\sqrt{3}} = \tan(75^\circ)}$

3.) (a) $\tan(\pi - \beta) = \frac{\tan\pi - \tan\beta}{1 + \tan\pi \cdot \tan\beta}$

$= \frac{0 - \tan\beta}{1 + 0 \cdot \tan\beta}$

$= \boxed{-\tan\beta}$

(b) $\cos(\pi - \beta) = \cos\pi\cos\beta + \sin\pi \cdot \sin\beta$

$= -1 \cdot \cos\beta + 0 \cdot \sin\beta$

$= \boxed{-\cos\beta}$

(c) $\sin(\beta - \pi) = \sin\beta\cos\pi - \cos\beta\sin\pi$

$= \sin\beta \cdot -1 - \cos\beta \cdot 0$

$= \boxed{-\sin\beta}$

4.) Find $\sin\theta$, $\cos\theta$, and $\tan\theta$

(a) $\cos(2\theta) = \frac{12}{13}$ when 2θ is in Q4.

$2\cos^2\theta - 1 = \frac{12}{13}$

$2\cos^2\theta = \frac{12}{13} + \frac{13}{13}$

$\frac{1}{2} \cdot 2\cos^2\theta = \frac{25}{13} \cdot \frac{1}{2}$

$\sqrt{\cos^2\theta} = \sqrt{\frac{25}{26}}$

$\cos\theta = \pm\frac{5}{\sqrt{26}}$

$\boxed{\cos\theta = -\frac{5\sqrt{26}}{26}}$

$\sin^2\theta + \left(\frac{-5}{\sqrt{26}}\right)^2 = 1$

$\sin^2\theta + \frac{25}{26} = 1$

$\sin^2\theta = \frac{1}{26}$

$\sin\theta = \frac{1}{\sqrt{26}}$

$\boxed{\sin\theta = \frac{\sqrt{26}}{26}}$

$\tan\theta = \dfrac{\frac{\sqrt{26}}{26}}{-\frac{5\sqrt{26}}{26}}$

$\boxed{\tan\theta = -\frac{1}{5}}$

(b) $\sin\left(\frac{\theta}{2}\right) = \frac{3}{5}$ when θ is in Q2.

$\sin\left(\frac{\theta}{2}\right) \to \left(\sqrt{\frac{1+\cos\theta}{2}}\right)^2 = \left(\frac{3}{5}\right)^2$

$\frac{1+\cos\theta}{2} = \frac{9}{25}$

$1 + \cos\theta = \frac{18}{25}$

$\cos\theta = \frac{18}{25} - 1$

$\boxed{\cos\theta = -\frac{7}{25}}$

$\sin^2\theta + \left(-\frac{7}{25}\right)^2 = 1$

$\sin^2\theta = 1 - \frac{49}{625}$

$\sin^2\theta = \frac{576}{625}$

$\boxed{\sin\theta = \frac{24}{25}}$

$\tan\theta = \dfrac{\frac{24}{25}}{-\frac{7}{25}}$

$\boxed{\tan\theta = -\frac{24}{7}}$

5.) (a) $\dfrac{\sin(2x-y)}{\sin(2x+y)} = \dfrac{\tan(2x)-\tan(y)}{\tan(2x)+\tan(y)}$ (work with the left side)

$\dfrac{\sin(2x)\cos(y)-\cos(2x)\sin(y)}{\sin(2x)\cos(y)+\cos(2x)\sin(y)}$ (mult. top & bottom by $\dfrac{1}{\cos(2x)\cos y}$)

$$\frac{\left[\sin(2x)\cos(y)-\cos(2x)\sin(y)\right]\cdot\frac{1}{\cos(2x)\cos(y)}}{\left[\sin(2x)\cos(y)+\cos(2x)\sin(y)\right]\frac{1}{\cos(2x)\cos(y)}} =$$

$$\frac{\dfrac{\sin(2x)\cos(y)-\cos(2x)\sin(y)}{\cos(2x)\cos(y)}}{\dfrac{\sin(2x)\cos(y)+\cos(2x)\sin(y)}{\cos(2x)\cos(y)}} =$$

$$\frac{\dfrac{\sin(2x)\cos(y)}{\cos(2x)\cos(y)}-\dfrac{\cos(2x)\sin(y)}{\cos(2x)\cos(y)}}{\dfrac{\sin(2x)\cos(y)}{\cos(2x)\cos(y)}+\dfrac{\cos(2x)\sin(y)}{\cos(2x)\cos(y)}} =$$

$$\boxed{\frac{\tan(2x)-\tan(y)}{\tan(2x)+\tan(y)}} = \text{same}$$

(b) $\dfrac{\cos(2x)}{\cos^2(x)} = \sec^2(x)-\tan^2(x)$

$\dfrac{\cos^2(x)-\sin^2(x)}{\cos^2(x)} =$

$\dfrac{\cos^2(x)}{\cos^2(x)}-\dfrac{\sin^2(x)}{\cos^2(x)} =$

$\boxed{1-\tan^2(x)}$ $\neq$ not an identity

5. (c) $\sin^2\left(\frac{x}{2}\right) = \dfrac{\sin^2(x)}{2+\sin(2x)\csc(x)}$

$= \dfrac{1-\cos^2(x)}{2+2\sin(x)\cos(x)\csc(x)}$

$= \dfrac{1-\cos^2(x)}{2+2\cos(x)}$

$= \dfrac{(1-\cos(x))(1+\cos(x))}{2(1+\cos(x))}$

$= \dfrac{1-\cos(x)}{2}$

$= \left(\sqrt{\dfrac{1-\cos(x)}{2}}\right)^2$

$= \sin^2\left(\frac{x}{2}\right)$

(d) $\sin(\beta-\pi) = -\sin(\beta)$

$\sin\beta\cos\pi - \cos\beta\sin\pi$

$\sin\beta\cdot -1 - \cos\beta\cdot 0$

$-\sin\beta$

6.) (a) $2\cos^3(\theta) = \cos(\theta)$

$2\cos^3\theta - \cos(\theta) = 0$

$\cos(\theta)(2\cos^2\theta - 1) = 0$

$\cos(\theta) = 0 \quad 2\cos^2\theta - 1 = 0$

$\theta = 90^\circ, 270^\circ$

$2\cos^2\theta = 1$

$\cos^2\theta = 1/2$

$\cos\theta = \sqrt{1/2} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = 45^\circ, 315^\circ$

(b) $\cos(3\alpha) = -\frac{\sqrt{3}}{2}$

If $0 \le \alpha \le \pi$ then $0 \le 3\alpha \le 3\pi$

$3\alpha = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}$

$\alpha = \frac{5\pi}{18}, \frac{7\pi}{18}, \frac{17\pi}{18}$

6. c.) $\cos(2x) = \frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4} + 2\pi k \text{ & } \frac{7\pi}{4} + 2\pi k$

$x = \frac{\pi}{8} + \pi k \text{ & } \frac{7\pi}{8} + \pi k$

or

$x = 22.5^\circ + 180^\circ k \text{ & } 157.5^\circ + 180^\circ k$

d.) $2\sin^2\beta + \sin\beta - 1 = 0$

$(x = \sin\beta)$

$2x^2 + x - 1 = 0$

$(2x - 1)(x + 1) = 0$

$x = 1/2 \qquad x = -1$

$\sin\beta = 1/2 \qquad \sin\beta = -1$

$\beta = \sin^{-1}\left(\frac{1}{2}\right) \text{ & } \beta = \sin^{-1}(-1)$

$\beta = 30^\circ + 360^\circ k \text{ & } \beta = 270^\circ + 360^\circ k$

and

$150^\circ + 360^\circ k$

OR

$\beta = \frac{\pi}{6} + 2\pi k \text{ and } \frac{5\pi}{6} + 2\pi k \text{ or}$

$\beta = \frac{3\pi}{2} + 2\pi k$

7.) $\sin\left(\frac{\theta}{2}\right) = -\frac{1}{2}$ θ is in Q3.

$\left(\sqrt{\frac{1-\cos\theta}{2}}\right)^2 = \left(-\frac{1}{2}\right)^2$

$\frac{1-\cos\theta}{2} = \frac{1}{4}$

$1 - \cos\theta = \frac{1}{2}$

$1 - 1/2 = \cos\theta$

$1/2 = \cos\theta$

$\theta = 60^\circ + 360^\circ k \text{ or } 300^\circ + 360^\circ k$

OR

$\theta = \pi/3 + 2\pi k \text{ or } 5\pi/3 + 2\pi k$

$\sin(2\theta) = \sin(120^\circ) = \boxed{\frac{\sqrt{3}}{2}}$

$\cos\left(\frac{\theta}{2}\right) = \cos(30^\circ) = \boxed{\frac{\sqrt{3}}{2}}$

$\cos(2\theta) = \cos(120^\circ) = \boxed{-\frac{1}{2}}$

$\tan(2\theta) = \tan(120^\circ) = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \boxed{-\sqrt{3}}$