MAC 1140 Precalculus – Mrs. Robertson

Name Solutions

Exam #3 – Chapter 4

No graphing or scientific calculators on problems 1 – 5. You may use a 4-function calculator

All <u>WORK</u> must be <u>SHOWN</u>. <u>CIRCLE</u> your final answers.

1. An investor wants to have a retirement account of $1,750,000 and estimates that her investment will grow at a rate of 2.5% compounded continuously for 40 years. What amount should be invested now to achieve this result? Round to the nearest penny if necessary.

$A = Pe^{rt}$

$1{,}750{,}000 = Pe^{(.025 \times 40)}$

$\frac{1{,}750{,}000}{e^{(.025 \times 40)}} = P$

$\boxed{P = \$643{,}789.02}$

2. Use your knowledge of transformations and inverse functions to help you <u>sketch</u> the following function (no graphing calculators). **Create a table of values** for the appropriate **exponential function** then switch it to **make a table of values for the parent logarithmic function and for the given log function below.**

$f(x) = log_3(x-1)$

exp. f(x)

$f(x) = 3^x$

x	f(x)
-1	1/3
0	1
1	3
2	9

parent log f(x)

$f(x) = \log_3(x)$

x	f(x)
1/3	-1
1	0
3	1
9	2

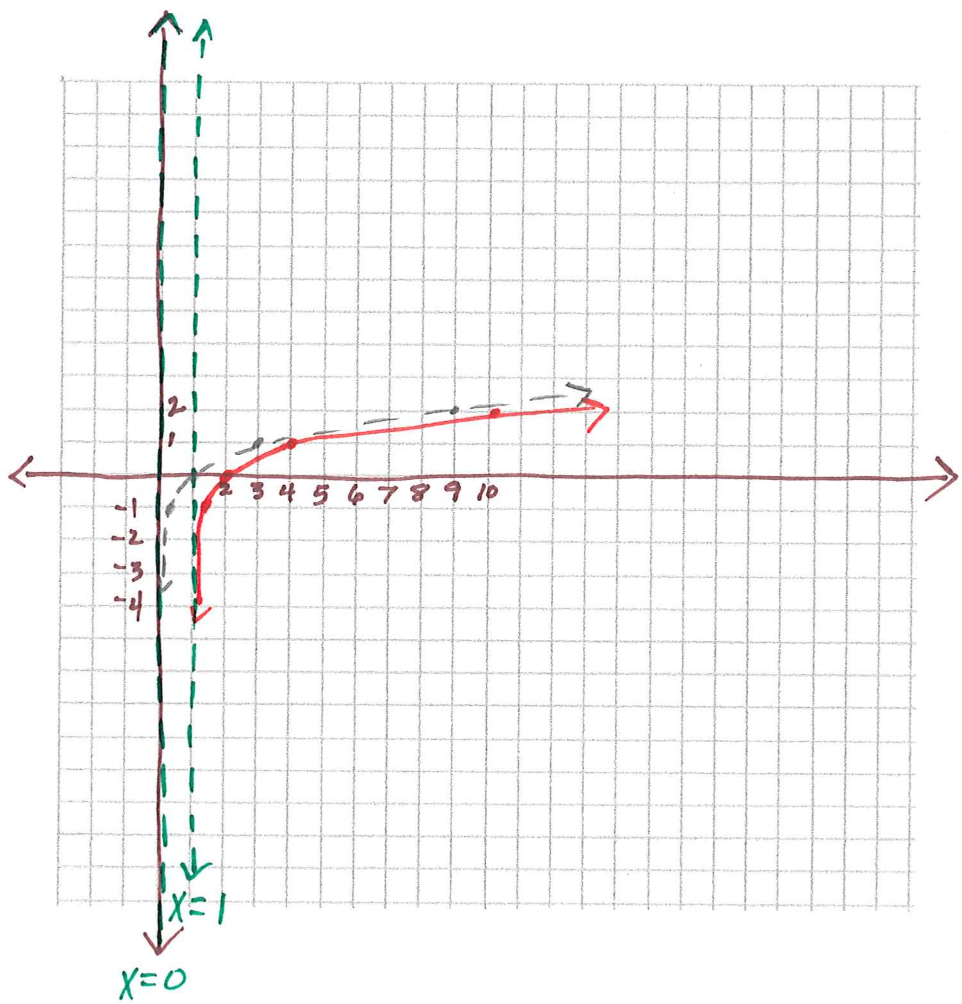
VA: x=0

given f(x)

$f(x) = \log_3(x-1)$

x	f(x)
1 1/3	-1
2	0
4	1
10	2

VA: x=1



3. Evaluate the following logarithmic and exponential expressions. You must show work!

a.) $\log_4 1024 = x$

$4^x = 1024$

$4^x = 4^5$

$\boxed{x = 5}$

b.) $27^{-\frac{4}{3}}$

$(\sqrt[3]{27})^{-4}$

3^{-4}

$\frac{1}{3^4} \rightarrow \boxed{\frac{1}{81}}$

4. Rewrite each expression as a sum or difference of logarithms.

a) $\log_2 \sqrt[5]{4x}$

$\log_2 (4x)^{1/5}$

$\frac{1}{5}\log_2(4x)$

$\frac{1}{5}\log_2(4) + \frac{1}{5}\log_2(x)$

$\frac{2}{5} + \frac{1}{5}\log_2(x)$

b) $\log_7 \left(\frac{2}{x}\right)^3$

$3\log_7\left(\frac{2}{x}\right)$

$3(\log_7(2) - \log_7(x))$

$\boxed{3\log_7(2) - 3\log_7(x)}$

5. The Richter scale rating of an earthquake of intensity (I) is given by $R = \log I - \log I_0$ where $I = 100{,}000 I_0$. Write the Richter scale as a single logarithm. (I = intensity)

$R = \log\left(\frac{I}{I_0}\right)$ $\quad R = \log\left(\frac{100{,}000\, \cancel{I_0}}{\cancel{I_0}}\right) \Rightarrow R = \log(100{,}000)$

$R = \log(10^5)$ $\boxed{R = 5}$

6. An earthquake had a Richter scale reading of 4.8. Find the intensity of the earthquake.

$4.8 = \log\left(\frac{I}{I_0}\right)$

$10^{4.8} = \frac{I}{I_0}$ $\quad \boxed{I = 10^{4.8} I_0}$

Calculators may be used for the remainder of the test BUT you must show all work!

Find the <u>exact</u> solution in terms of the common or natural logarithms, then give the decimal approximation to 3 decimal places. Do not use change of base rule.

7. $4^{3x} = 11$

$\ln(4^{3x}) = \ln(11)$

$3x \ln(4) = \ln(11)$

$\boxed{x = \frac{\ln(11)}{3\ln(4)}}$

8. At 8:00 am a Petri dish contains 200 bacteria. The doubling time for bacteria is 1.5 hours. Answer the following questions to the nearest minute. Round the rate to 4 decimal places.

$400 = 200e^{1.5r}$ $\quad \ln(2) = 1.5r \quad r = .4621$

$2 = e^{1.5r}$ $\quad \frac{\ln(2)}{1.5} = r$

a) At what time will there be 1000 bacteria in the dish?

$1000 = 200e^{.4621t}$ $\quad \ln(5) = .4621t$ $\quad t = 3.482877775\ldots$ hrs

$5 = e^{.4621t}$ $\quad \frac{\ln(5)}{.4621} = t$ $\quad t = 3$ hrs 29 min

8:00 AM + 3:29

$\boxed{11:29 \text{ AM}}$

b) At what time were there only 50 bacteria in the dish?

$50 = 200e^{.4621t}$ $\quad \ln(.25) = .4621t$ $\quad t = -3$ hrs

$.25 = e^{.4621t}$ $\quad t = -2.999987797$

8:00 AM − 3:00

$\boxed{5:00 \text{ AM}}$

c) At what time will there be 3,000 bacteria in the dish?

$3000 = 200e^{.4621t}$ $\quad \ln(15) = .4621t$

$15 = e^{.4621t}$ $\quad t = 5.860312056 \rightarrow t = 5$ hrs 52 min

8:00 AM + 5:52

13:52 → $\boxed{1:52 \text{ PM}}$

d) How many bacteria will be present at noon?

Noon is 4 hours from 8:00 AM.

$A = 200e^{.4621 \times 4}$ $\quad A = 1{,}269.93039$ or $\boxed{A = 1{,}270 \text{ bacteria}}$

9. Newton's law of cooling states that when a warm object is placed in a cooler environment or a cooler object is placed in a warmer environment then the difference between the two temperatures decreases exponentially.

If D_0 is the initial **difference** in temperatures, then the difference D at the time t is modeled by the formula.

Environment temp − body temp

$$D = D_0e^{kt}$$

$t = 1$ hr

$D_0 = 45° - 80° \rightarrow -35°$ (4:00 AM)

$D_1 = 45° - 76° \rightarrow -31°$ (5:00 AM)

A detective discovered a body in a vacant lot at 4:00 am and found that the body temperature was 80 degrees F. The county coroner examined the body at 5:00 am and found the body temperature to be 76 degrees. Assuming that the body temperature was 98 degrees when the person died and that the air temperature was a constant 45 degrees all night, what was the approximate time of death? Round the rate to $\boxed{\text{4 decimal places}}$ and round the time to the nearest minute.

To find K − (rate)

$-31 = -35e^{K \cdot 1}$

$\frac{31}{35} = e^{1K}$

$\ln(31/35) = K$

$K = -.1214$

To find t (time of death)

$D = D_0e^{-.1214t}$

$(45-80) = (45-98)e^{-.1214t}$

$-35 = -53e^{-.1214t}$

$\frac{35}{53} = e^{-.1214t}$

$\ln\left(\frac{35}{53}\right) = -.1214t$

$t = 3.417988897$

$t = 3$ hrs 25 min since time of death

3:60
4:00 AM
− 3:25
0:35 or

$\boxed{12:35 \text{ AM TOD}}$