MAC 2313 Calc III Practice Test #2 Name: Robertson - Solutions

Mrs. Robertson Test #2 will be Mar. 3rd. 2/24/20

1. For the function $f(x,y) = \sqrt{2x^2 - 3x} + \sqrt{y^2 - 9}$

   a. Evaluate $f(-\frac{3}{2}, 5) = \sqrt{2(-\frac{3}{2})^2 - 3(-\frac{3}{2})} + \sqrt{5^2 - 9} = \boxed{7}$

   b. Find the domain of the function values of x where $2x^2 - 3x \geq 0$ and values of y where $y^2 - 9 \geq 0$ ~~$2x^2 - 3x + y^2 - 9 \geq 0$~~

2. Find $\lim_{(x,y)\to(1,1)} \frac{2xy}{x^2+2y^2}$

direct substitution $\lim_{(x,y)\to(1,1)} \frac{2xy}{x^2+2y^2} = \frac{2(1)(1)}{1^2 + 2(1)(1)^2} = \boxed{\frac{2}{3}}$

3. Show that $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2}$ does not exist by demonstrating that you get different limiting values when (x,y) approaches $(0,0)$ along different paths. Hint: Consider linear paths of the form $y = mx$.

$\lim_{(x,y)\to(0,0)}$ (along x-axis) $\frac{2x \cdot 0}{x^2+0} = \frac{0}{x^2} = \boxed{0}$

$\lim_{(x,y)\to(0,0)}$ (along y-axis) $\frac{2(0)y}{0+2y^2} = \frac{0}{2y^2} \; \boxed{0}$

$\lim_{(x,y)\to(0,0)}$ (along $y=x$) $= \frac{2(x)(x)}{x^2+2x^2} = \frac{2x^2}{3x^2} = \boxed{\frac{2}{3}}$

Since all limits are not equal - the limit doesn't exist.

4. For the function $f(x,y,z) = x^3yz^2 + 2yz$, find each of the following:

   a. $f_x(x,y,z) = \boxed{3x^2yz^2}$ $\qquad f_{xx} = 6xyz^2$

   b. $f_y(x,y,z) = \boxed{x^3z^2 + 2z}$

   c. $f_z(x,y,z) = \boxed{2x^3yz + 2y}$

   d. $f_{xz}(x,y,z) = \boxed{6x^2yz}$

   e. $f_{yy}(x,y,z) = \boxed{0}$

   f. $f_{zy}(x,y,z) = \boxed{2x^3z + 2}$

   g. $f_{xxy}(x,y,z) = \boxed{6xz^2}$

5. For the function $z = \ln(x - 2y)$, find the equation of the tangent plane at the point $(3,1,0)$. Give your answer in point-slope form, $z = m_1(x - x_0) + m_2(y - y_0) + z_0$

$f_x = \frac{1}{x-2y} \qquad f_y = \frac{-2}{x-2y}$

$f_x(3,1,0) = 1 \qquad f_y(3,1,0) = -2$

$z = 1(x-3) + (-2)(y-1) + 0$

$\boxed{z = (x-3) - 2(y-1)}$ OR

$\boxed{z = x - 2y - 1}$

6. For the function $f(x, y) = x^2 + xy + 3y^2$

$f_x = 2x + y \quad f_y = xy + 6y$

a. Find the linearization, $L(x, y)$, at the point $(1, 1)$

$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$L(x,y) = 5 + 3(x-1) + 7(y-1)$ $\boxed{L(x,y) = 3x + 7y - 5}$

$\Delta x = .1 \quad \Delta y = -.2$

b. Use the differential of f, df, to estimate the change in f when (x, y) varies from $(1, 1)$ to $(1.1, 0.8)$

$df = (2x+y)(.1) + (xy - 6y)(-.2)$ $\boxed{df = -1.1}$

$= (3)(.1) + (7)(-.2)$

$= .3 - 1.4$

$\Delta f = f(1.1, 0.8) - f(1,1)$

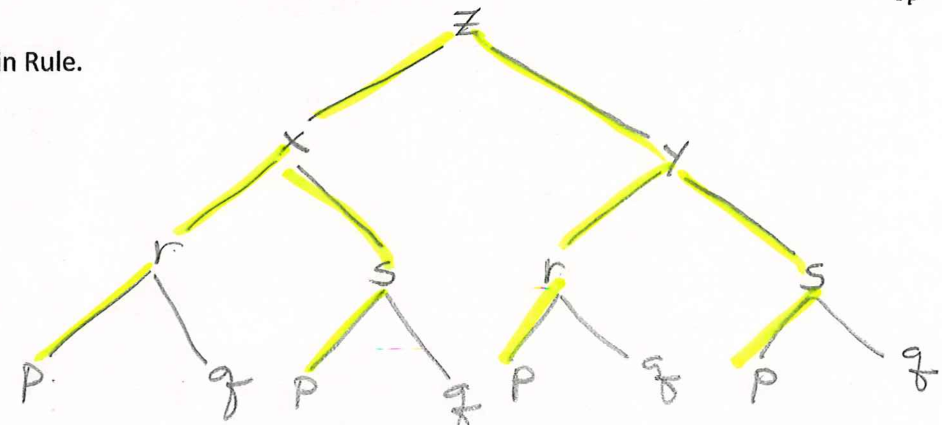
$= 4.01 - 5$

$\boxed{\Delta f = -.99}$

$df \approx \Delta f$ when rounded to the nearest whole number

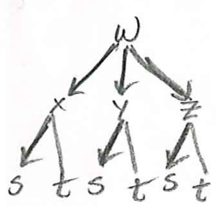
7. Suppose z is a function of x and y, and x and y are functions of r and s, and r and s are functions of p and q.

Draw a tree diagram to represent the chain of dependency, and then write out the formula for $\frac{\partial z}{\partial p}$ that would be dictated by the Chain Rule.



$$\boxed{\frac{\partial z}{\partial p} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r}\frac{\partial r}{\partial p} + \frac{\partial z}{\partial x}\frac{\partial x}{\partial s}\frac{\partial s}{\partial p} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}\frac{\partial r}{\partial p} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}\frac{\partial s}{\partial p}}$$

8. Let $w = x^2 + 2y^2 + 3z^2 + 2xy + 3yz - xz$. Suppose $x = 2s + t$, $y = t^2 - s^2$ and $z = \sin(s) + \cos(t)$. Use the Chain Rule to find $\frac{\partial w}{\partial s}$. You may leave your formula in terms of x, y, z, and s. (You are not required to get the answer entirely in terms of s). It is not necessary to distribute out your answer.



$\frac{\partial w}{\partial x} = 2x + 2y - z \qquad \frac{\partial w}{\partial y} = 4y + 2x + 3z \qquad \frac{\partial w}{\partial z} = 6z + 3y - x$

$\frac{\partial x}{\partial s} = 2 \qquad \frac{\partial y}{\partial s} = -2s \qquad \frac{\partial z}{\partial s} = \cos(s)$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

$$\boxed{\frac{\partial w}{\partial s} = (2x + 2y - z)(2) + (4y + 2x + 3z)(-2s) + (6z + 3y - x)\cos(s)}$$

$F(x,y,z) = xz - x\ln(y) - z^2$

9. Given the equation $xz + x\ln(y) = z^2$, treating z as an implicit function of x and y, find $\frac{\partial z}{\partial x}$. You may either use Implicit Differentiation or the Implicit Function Theorem.

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} \qquad \frac{\partial F}{\partial x} = z - \ln(y) \qquad \frac{\partial F}{\partial z} = x - 2z$$

$$\boxed{\frac{\partial z}{\partial x} = -\frac{z - \ln(y)}{x - 2z}}$$

10. For the function $f(x,y) = x^2\ln(y)$, find each of the following:

a. $\nabla f(x,y) = \left\langle 2x\ln(y), \frac{x^2}{y} \right\rangle$

b. $\nabla f(3,1) = \langle 0, 9 \rangle$

c. The directional derivative of f at the point $(3,1)$ in the direction of the vector $\langle -5,\ 12 \rangle$

$|v| = \sqrt{(-5)^2 + 12^2} = \sqrt{169} = 13$

$u = \left\langle \frac{-5}{13}, \frac{12}{13} \right\rangle$

$D_u f(3,1) = \nabla f \cdot u = \langle 0, 9 \rangle \cdot \left\langle \frac{-5}{13}, \frac{12}{13} \right\rangle = \boxed{\frac{108}{13}}$

d. The maximum rate of change of f (i.e., the maximum value of its directional derivative at the point $(3,1)$.

max. rate of change $= |\nabla f| = \sqrt{0^2 + 9^2} = \boxed{9}$

11. For the level surface $xy^2z^3 = 8$, find the equation of the tangent plane at the point $(2,2,1)$. Give your answer in standard form, $Ax + By + Cz = D$

$f_x = y^2z^3 \quad f_y = 2xyz^3 \quad f_z = 3xy^2z^2$

$f_{(x)}(2,2,1) = 4 \quad f_y(2,2,1) = 8 \quad f_z(2,2,1) = 24$

$4(x-2) + 8(y-2) + 24(z-1) = 0$

$4x - 8 + 8y - 16 + 24z - 24 = 0$

$\boxed{4x + 8y + 24z = 48}$